

ARTIGO <https://doi.org/10.22481/praxisedu.v15i36.5937>**“MATHEMATICA” IN TEACHING STUDENTS MATHEMATICS****"MATHEMATICA" EN LA ENSEÑANZA DE MATEMÁTICAS A LOS ESTUDIANTES****“MATHEMATICA” NO ENSINO DE MATEMÁTICA PARA ESTUDANTES***Anton N. Karamyshev*

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Resumo: A relevância do tópico do artigo se deve ao processo de modernização do ensino superior de matemática na Rússia, que levou a uma mudança significativa nos currículos e a necessidade de procurar formas e formas de treinamento que permitam aos alunos aprender o necessário material dentro do tempo concedido para o estudo, enquanto obtém a quantidade máxima necessária de habilidades, conhecimentos e competências. O objetivo do artigo é justificar os caminhos e princípios do desenvolvimento e implementação de novas tecnologias pedagógicas e da informação no processo educacional, a organização da formação profissional de estudantes em áreas técnicas baseadas na integração da matemática e da computação. O método principal do estudo desse problema é a análise metodológica e a síntese subsequente, que, analisando o conteúdo didático das seções em matemática e as possibilidades do ambiente matemático computacional chamado Mathematica, revelam os métodos e formas necessários para desenvolver e usar modernas tecnologias de computador na educação matemática de estudantes de engenharia. Está provado que uma das principais ferramentas para implementar os métodos de solução do problema indicado deve ser considerada um computador, a saber, o ambiente matemático chamado Mathematica e os princípios básicos de sua implementação sistêmica no processo educacional da universidade. Os materiais do artigo podem ser úteis para professores de disciplinas matemáticas de instituições de ensino superior, os programas de computador e os produtos de software pedagógico criados no Mathematica podem servir como modelos para o desenvolvimento de produtos de software pedagógico semelhantes.

Palavras-chave: Simulador; Livro didático de informática; Sistema matemático computacional; Informatização da educação matemática; Tecnologia da informação.

Abstract: The relevance of the topic of the article is due to the process of modernization of higher mathematical education in Russia, which has led to a significant change in curricula and the need to look for ways and forms of training that would allow students to learn the necessary material within the time granted for studying, while obtaining the maximum necessary amount of skills, knowledge, and competencies. The objective of the article is to justify the ways and principles of the development and implementation of new pedagogical and information technologies in the educational process, the

organization of professional education of students in technical areas based on the integration of mathematics and computer science. The leading method of the study of this problem is the methodological analysis and subsequent synthesis, which, by analyzing the didactic content of the sections in mathematics and the possibilities of the computer mathematical environment called Mathematica, reveals the necessary methods and ways of developing and using modern computer technologies in the mathematical education of engineering students. It is proved that one of the main tools for implementing the methods for solving the indicated problem should be considered a computer, namely, the mathematical environment called Mathematica, and the basic principles of its systemic implementation in the educational process of the university have been identified. The materials of the article may be useful to teachers of mathematical disciplines of higher educational institutions, the computer programs and pedagogical software products created in Mathematica can serve as models for the development of similar pedagogical software products.

Keywords: Simulator; Computer textbook; Computer mathematical system; Computerization of mathematical education; Information technology.

Resumen: La relevancia del tema del artículo se debe al proceso de modernización de la educación matemática superior en Rusia, que ha llevado a un cambio significativo en los planes de estudio y la necesidad de buscar formas y formas de capacitación que permitan a los estudiantes aprender lo necesario material dentro del tiempo otorgado para estudiar, mientras se obtiene la cantidad máxima necesaria de habilidades, conocimientos y competencias. El objetivo del artículo es justificar las formas y principios del desarrollo e implementación de nuevas tecnologías pedagógicas y de información en el proceso educativo, la organización de la educación profesional de los estudiantes en áreas técnicas basadas en la integración de las matemáticas y la informática. El método principal para el estudio de este problema es el análisis metodológico y la síntesis posterior, que, al analizar el contenido didáctico de las secciones en matemáticas y las posibilidades del entorno matemático informático llamado Mathematica, revela los métodos y formas necesarios de desarrollar y utilizar tecnologías informáticas modernas en la educación matemática de estudiantes de ingeniería. Está comprobado que una de las principales herramientas para implementar los métodos para resolver el problema indicado debe considerarse una computadora, a saber, el entorno matemático llamado Mathematica, y se han identificado los principios básicos de su implementación sistémica en el proceso educativo de la universidad. Los materiales del artículo pueden ser útiles para maestros de disciplinas matemáticas de instituciones de educación superior, los programas de computadora y los productos de software pedagógico creados en Mathematica pueden servir como modelos para el desarrollo de productos de software pedagógico similares.

Palabras clave: Simulador; Libro de texto informático; Sistema matemático informático; Informatización de la educación matemática; Tecnología de la información.

Introduction

Information technology plays an important role in the modern world. It penetrates the life of every person, and information competence is one of the most important indicators of a person's level of education. The requirements of information competence are laid down in the new Federal state standards of higher professional education of the Russian Federation.

Currently, there are many contradictions between the existing training system, which relies on limited content standards, a lesson training system, and the enormous capabilities of modern computer technology and computer technology. It is obvious that improvement of the training system requires, along with traditional teaching methods, introducing new pedagogical and information technologies. Today, higher education institutions are dominated by collective forms of learning that do not take into account individual differences in the assimilation and application of new knowledge by students. Mathematics, as an academic subject, differs from other disciplines taught at the university by a number of features:

- 1) the high complexity of its study;
- 2) the frequent occurrence of difficulties in the cognitive component of learning;
- 3) a significant influence of the individual characteristics of the trainees on the assimilation of the subject;
- 4) the level of cognitive motivation and school mathematical training of subjects of education.

Of course, the result is the that it is necessary to look for such methods and forms of training that would allow students to learn the necessary material in a limited curriculum, while obtaining the maximum amount of skills and knowledge. Of particular relevance is the use of specially designed multimedia resources and the optimal organization of students' independent activities. Speaking about the formation of students' independence, we need to pay attention to two closely related tasks. The first is that students need to instill a need for independence in cognitive activity, to teach them to independently master knowledge, and to develop a greater interest in obtaining new knowledge, in continuous self-education, and to form their own worldview; the second is that teacher needs to teach them to independently use the available knowledge in practical activities.

Using specially designed multimedia resources makes it possible to implement such learning objectives as the development of algorithmic, spatial, intuitive, and creative thinking, the formation of optimal decision-making methods based on several possible options. All this leads to the fact that the students release their personal time, therefore, the learning process is accelerated and acquires new features.

We also note that individual work related to the study of educational literature published in print media retains an important link in the independent work of students in general but in modern conditions independent work can be intensified with the help of training programs, a test system, and information databases. Educational computer programs exercise

one of the main prospects in the development of new information technologies, with the help of which it would be possible to teach subjects related to mathematics. They make it possible to see illustrations from the main concepts of a mathematics course at a level that provides a greater advantage compared to the old traditional methods of teaching the subject. A substantial activation of the student's cognitive activity, a combination of imaginative thinking and verbal-logical mechanisms, as well as an increase in the visibility of the information received are laid in their base. However, the work of compiling computer-based training and educational-controlling programs requires significant time resources and special knowledge. One should try to create some standard shells for such programs where teachers of mathematics could invest their didactic content in; the article describes just such an approach (Zaitseva & Gubochkina, 2014; Zaitseva, 2016; Kapustina et al., 2014; Kapustina et al., 2015).

Main Part

To date, several effective computer mathematical systems have been developed to simplify the computer implementation of mathematical algorithms and methods for solving quite complex problems for users without going into the intricacies of programming. One of such mathematical system is Mathematica. Mathematica computer mathematical system can be used in free mode both for performing laborious calculations and as an ideal tool for self-checking, finding and correcting errors in solving tasks.

We can distinguish two approaches to practical training in the methodology of teaching mathematics with the use of Mathematica:

- 1) without the use of additional pedagogical software products (PSW),
- 2) with the use of additional PSW.

The first approach assumes that students work in the Mathematica system in free mode, while showing independence and initiative in solving problems. The second approach, as a rule, requires the teachers themselves to have in-depth knowledge of the Mathematica system, which they use in teaching, the ability to program and the availability of time to prepare the PSW.

We shall consider the teaching methods for studying the section "Harmonic analysis (Fourier series)" in the course of mathematics for the technical specialties of the university. We briefly describe the methodological features of using the Mathematica system in the study of this section.

First of all, when studying the topic “Fourier series”, the Mathematica system can be applied in practice for in-depth study of this material.

Mathematica can be used in free mode both for performing laborious calculations, and as an ideal tool for self-checking, finding and correcting errors in solving problems.

Example.

Expand in a Fourier series 2π - a periodic function defined by the formula $f(x) = x$, $x \in [-\pi, \pi]$.

Decision.

We introduce the function

In [1].:= f[x_]:= x

We plot the function

In [2].:= Plot[f[x], {x,-π,π}]

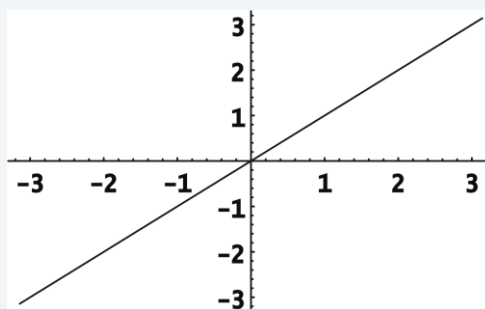


Fig. 1. The function graph $f(x) = x$

The function $f(x) = x$ is odd; it is known that if an odd function $f(x)$ is expanded in a Fourier series, then the product $f(x) \cdot \cos(kx)$ is also an odd function but $f(x) \cdot \sin(kx)$ is an even

$$\left. \begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = 0, \\ a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx dx = 0, \\ b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx dx = \frac{2}{\pi} \int_0^{\pi} f(x) \sin kx dx \end{aligned} \right\}$$

function; therefore, if

i.e., the Fourier series of an

odd function contains only sines, then we obtain

$$f(x) = \sum_{k=1}^{\infty} b_k \sin kx$$

We calculate the Fourier coefficients of the function $f(x)$:

$$\text{In}[3]:= \text{b}[k_]=\frac{2}{\pi}\int_0^{\pi}f[x]\text{Sin}[k x]dx//\text{Simplify}$$

$$\text{Out}[3]:= \frac{-2k\pi\cos[2\pi]+2\sin[k\pi]}{k^2}$$

Using formulas $\cos(k\pi)=(-1)^k$, $\sin(k\pi)=0$ we simplify the result

$$b_k = \frac{-2 \cdot k \cdot \pi \cdot (-1)^k + 2 \cdot 0}{k^2} = \frac{2}{k}(-1)^{k+1}$$

Therefore, the following equality is true for $\forall x \in \mathbf{R}$, $x \neq \pm(2k-1)\pi$ ($k \in \mathbf{N}$)

$$f(x) = 2 \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin(kx)}{k} \quad (1)$$

At the discontinuity points $x = \pm\pi$, the sum $S(x)$ of the Fourier series (1) of the function $f(x)$

is found by the formula $S(-\pi) = S(\pi) = \frac{1}{2} \left[\lim_{x \rightarrow -\pi+0} x + \lim_{x \rightarrow \pi-0} x \right] = 0$; at all other discontinuity

points $x = \pm(2k-1)\pi$, $n \in \mathbf{N}$, the sum of the Fourier series (1) of the function $f(x)$ is also equal to zero.

We calculate the partial sums of the Fourier series (1)

$$\text{In}[4]:= \text{s}[n_ , x_] := \sum_{k=1}^n \text{b}[k] \text{Sin}[k x]$$

We plot some partial sums and the original function

$$\text{In}[5]:= \text{Plot}[\{\text{s}[2, x], \text{s}[5, x], \text{s}[10, x], f[x]\}, \{x, -\pi, \pi\}]$$

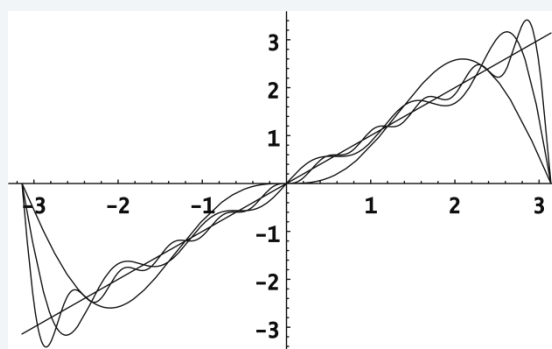


Fig. 2. Graph of the function $f(x) = x$ and three partial sums for $n = 2, 5, 10$

Based on this example, students can draw the following conclusion: the more n is in the partial sum of the Fourier series, the closer is its graph to the graph of the original function. According to the graph, the meaning of the approximation of the function by its Fourier series is clearly visible.

This task is undoubtedly time-consuming if solved without computer assistance.

Mathematica allows creating and introducing PSW in the learning process, such as a simulator, a computer textbook, and a computer textbook with elements of the simulator.

The simulator, developed and used in Mathematica, is a pedagogical software which action is based on specific program procedures of the built-in programming language of the Mathematica system. The simulator is a training and monitoring program, which is supposed to be used both in the classroom process and in students' independent work.

We shall consider the work of the simulator on the topic of "Fourier series".

The example of expanding a function in a Fourier series contains a record: "Next, select the hidden cell with the left mouse button, press Shift + Enter in turn and follow the further instructions". After this instruction has been fulfilled, a dialog box appears with the inscription what you need to write in it, and the simulator enters a dialogue with the user (Figure 3).

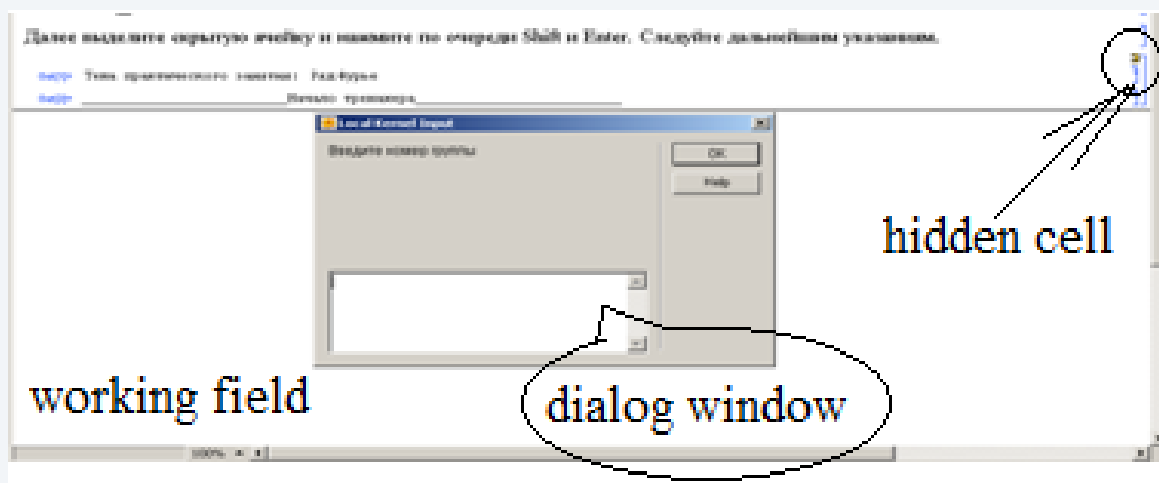


Fig. 3. Simulator in operation

The surname, first name, student group number, and example condition are entered into the dialog box. After introducing the conditions of the example, you need to solve it and write the received answer in the dialog box. All actions performed are reflected in the working field. If the entered result is correct, the entry "Well done! The number of errors is 0" Will

appear in the working field and the student proceeds to follow further instructions; otherwise, another entry will appear in the working field: "You made a mistake. You have only 2 attempts left". In this case, it is possible to independently find an error in the solution of the example. It is necessary to re-resolve the task and write the corrected value in the dialog box; if it is correct, the entry "Well done. The number of errors is 1" will appear, and you can proceed to further instructions. Otherwise there will be a record "You made a mistake. You have only 1 attempt left", and you can again try to find the error, etc. In total, three errors can be made, after which the simulator displays the correct answer and the number of errors made. At the end of the work, the simulator provides the opportunity to print the results for verification and evaluation by the teacher.

The fundamental difference between the simulator and test tasks is that the simulator introduces an intermediate solution to the example and not just the answer. For example, when passing through the simulator, performing the task of expanding the function in a Fourier series, it will be necessary to determine the parity, the oddness of the function, calculate and enter the coefficients of the Fourier series, calculate the n -sum of the Fourier series, in accordance with the option performed, 14 errors can be made in total (Zaitseva, 2014).

Since the simulator program code is contained in one hidden cell, you can type any number of the same type of examples, which is important.

In this section, you can make a computer textbook or computer textbook with elements of the simulator. As you know, "... a computer (electronic) textbook is a computer-based learning tool for basic training in a certain subject (discipline), the content of which is characterized by relative completeness and is presented in the form of a textbook (book) on an electronic medium" (Kapustina & Zaitseva, 2006).

We have created a computer textbook with an element of the simulator on the topic "Fourier series", which contains the following sections:

Section 1. Historical background.

Section 2. Theoretical background.

Section 3. Simulator.

Section 4. Reference background.

Section 2 presents theoretical positions: the general form of the Fourier series, theorems on the decomposability of a periodic function into a Fourier series, the convergence of a Fourier series, the determination of the coefficients of a Fourier series, various types of

formulas for determining the coefficients of a Fourier series $2\pi, 2l$ – periodic functions, for functions given with arbitrary period, both in sine and in cosine of multiple angles. The solutions are provided to a wide range of problems on the expansion of specific functions in a Fourier series, which is based on the use of the Mathematica system, and various approximations of the Fourier series are also demonstrated to the student. In this case, the results contained in the fourth section are used. A computer textbook with simulator elements on the subject of “Fourier series” is organized as a multi-level one, using a system of hyperlinks you can freely move from one section to another.

A computer textbook and a computer textbook with elements of a simulator for teaching mathematics can be used to organize students' independent work, to test and control knowledge at any stage of training, as well as to reorient classrooms with students to a higher creative level, instructing the computer on the initial stage of training. We should note that the organization of the educational process, based on the latest technologies in teaching mathematics and the widespread use of the Mathematica computer mathematical system, is consistent with the idea of integrating mathematics and computer science in the professional and mathematical training of students (Kapustina & Zaitseva, 2006). As we can see, the possibilities of using the computer mathematical system Mathematica in teaching are very wide.

Methods

We used the following methods:

- 1) theoretical - analysis of the scientific literature on mathematical, methodological, and special aspects related to this topic;
- 2) empirical - conversations, pedagogical observations, questioning of students and teachers;
- 3) programming methods in WolframLanguage.

Results And Discussion

Methods and techniques of application in the educational process of the university computer mathematics system Mathematica are formulated. At the same time, the authors relied on their own development, patented by copyright certificates.

Questioning and interviews show that, in general, teachers and students have positive opinion about the appropriateness of using Mathematica in the educational process.

Summary

A generalization of this experience allows us to draw the following conclusions. Using Mathematica as a means for conducting auxiliary calculations in the process of solving educational problems allows us to realize the following methodological goals:

- 1) increasing motivation in the study of mathematics among students of technical specialties;
- 2) increasing the level of visibility of practical exercises;
- 3) reducing the study time, exemption from routine calculations; due to the release of time, it becomes possible to focus on the ideological side of the issue under study;
- 4) providing the opportunity to introduce into the educational process tasks that were not previously solved in practical classes, that is, expanding the didactic content of lecture and practical classes (the embodiment of the principle of new tasks);
- 5) the use of a computer textbook with simulator elements as a means of preparing a student for the qualified use of a computer in educational activities, and (or) as a training and monitoring program during practical classes or at home for better organization of independent work of full-time, distance and distance students;
- 6) the facilitation of the work of teachers of mathematics who can adapt the existing simulator to their methodological needs, compiling it according to the same principle (using the same basis of program code) but giving a different (author's) didactic content to use a new simulator for studying various topics and branches of mathematics, where a numerical characteristic of the sought quantities is required, for example: "Matrices and actions on them", "Calculation of the limits of functions", etc (Zaitseva, 2005; Zaitseva, 2017; Engelbart & Watson, 1976; Gray et al., 1996).

Conclusions

The use of Mathematica in the educational process, of course, cannot replace the traditional forms of teaching but it will effectively complement and enrich them. Its competent

use will help to significantly intensify the educational process, to illuminate the phenomenon or object being studied from different angles. It is also important to prepare the student for the qualified use of the computer in educational activities, to make the learning process more attractive and interesting.

Acknowledgments

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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Recebido em: 30 de outubro de 2019
Aprovado em: 23 de novembro de 2019
Publicado em: 09 de dezembro de 2019