Linear programming problems in high school using GeoGebra

Gonçalo Renildo Lima Cerqueira
Department of Exact and Technologic Sciences
State University of Southwest Bahia
Vitoria da Conquista,Brazil
goncalorenildo@uesb.edu.br

Velton Cardoso Pires
State University of Southwest Bahia
Vitoria da Conquista,Brazil
vellpires@gmail.com

Daniel Silva Campos
State University of Southwest Bahia
Vitoria da Conquista,Brazil
danyelmatematica@gmail.com

Abstract—**Many high school teachers when teaching mathematical content, face questions from their students about the applicability of these contents. We will show in this paper that it is possible to answer some of these questions by relating some of this content to optimization problems, which are modeled as linear programming problems and solved using GeoGebra as a tool.**

Index Terms—**Optimization. Linear Programming. High School. GeoGebra**

I. Introduction

The reform of high school that has been discussed in the National Congress and among several brazilian educators, has caused some uncertainties among teachers and students as to its real effectiveness in better qualification of teaching, but it is unanimous among all that high school needs to undergo changes. Domingues [3] see urgency in the reform of secondary education, but he thinks it is important that it be better discussed in all instances related to education. The aim of this work is to present a proposal to address some mathematical content from high school that is attractive to students, that arouses the same interest in the subject and enables the teacher to apply it well.

According to Silva [10] the 2007 School Census, analyzed by the National Institute of Educational Studies and Research Anísio Teixeira (Inep / MEC), dropping out of school among young people is alarming. Of the 3.6 million who enroll in high school, only 1.8 million complete this degree. The evasion rate is 13.3% in high school against 6.7%, from fifth to eighth grade, and 3.2%, from first to fourth grade. Brazil currently has 8.3 million high school students enrolled in 24,000 schools, 17,000 of which are public and half of these, according to the Ministry of Education, do not finish their studies. With an index of 24.3%, Brazil has the third highest dropout rate among the 100 countries with the highest HDI (Human Development Index), only behind Bosnia Herzegovina (26.8%) and the Islands of São Cristovam and Névis, in the Caribbean (26.5%). In Latin America, only Guatemala (35.2%) and Nicaragua (51.6%) have higher dropout rates ". Gonçalves [6] conducted a survey among high school students and identified that between 48.3% and 61.7% of the interviewees had difficulties in learning mathematics, which is certainly one of the reasons why many students leave school. It was motivated by the difficulties found by high school teachers in relating some mathematical content with practical applications of daily life for their students, which we decided to present in this article, a proposal to use GeoGebra as a computational tool to solve geometrically linear programming problems, identifying the associated mathematical content. Martins [7] points out that it is necessary to connect some content taught in basic education to each other and bring them closer to the experience of our students and that the concepts of linear programming can provide such an approach.

II. Linear Programming Problem

For the mathematical formulation of a linear programming problem (LP) we must determine the decision variables involved, the objective function and model constraints. Source Bazaraa [1]. The decision variables arise naturally from the understanding of the problem to be studied. They will point out the decisions to be made according to the nature of the proposed problem, which may involve production, investments, distribution and selection of products and services, location etc. Once the possible decisions have been established, it must be very clear in the problem what the objective is to be achieved, what is intended to be optimized with the possible decision making previously prepared. In the literature, such problems are cited as optimization problems (Freitas [4], Dantas [2]), which can be associated with maximizing profits, minimizing costs or losses, reducing time, choosing the best location, best distribution of goods, etc. Finally, to complete the mathematical formulation, the restrictions of the problem are added to the model, which are conditions imposed on the decision-making process, usually associated with limitations of available resources, such as raw material, labor, production time, etc. The objective function is linear and the constraints are represented by linear equations or inequalities. Second Goldbarg [5], to formulate the mathematical model one can consider $\mathbf{x} = [x_1 \ x_2 \ ... \ x_n]$ the matrix of decision variables, where x_j is the decision variable representing the amount of the resource of the type j available to $j = 1,...,n$. $\mathbf{c} = [c_1 \ c_2 \ ... \ c_n]$ indicates the costs involved. $\mathbf{b} = [b_1 \ b_2 \ ... \ x_m]$ the matrix of independent terms, in which b_i is the available quantity of the resource of type i; and $A = a_{ij}$ an order matrix $(m \times n)$ on what a_{ij} is the amount of type i resource associated

with decision variable j. for $i = 1, ..., m$; $j = 1, ..., n$. The complete model is presented below.

$$min \ z = c_1 x_1 + c_1 x_2 + \cdots + c_n x_n$$
$$subject \ to:$$
$$a_{11} x_1 + a_{12} x_2 + \cdots a_{1n} x_n \le b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots a_{2n} x_n \le b_2 \qquad (1)$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots a_{mn} x_n \le b_m$$
$$b_i \ge 0; x_j \ge 0$$

With the notations defined, the model takes the following matrix form.

$$min \ z = \mathbf{cx}$$
$$subject \ to: \qquad (2)$$
$$A\mathbf{x} \le \mathbf{b}$$
$$\mathbf{x} \ge \overrightarrow{0}; \mathbf{b} \ge \overrightarrow{0}$$

III. THE GEOMETRY OF A LINEAR PROGRAMMING PROBLEM

One way to solve a linear programming problem is to consider some important properties of a regular polygon. For this, consider the following example:

Site problem: Manoel, after some years of work, decides to buy a 90 hectare farm to plant coffee and cocoa. Each hectare of coffee generates a profit of $R\$400,00$, and each hectare of cocoa, returns $R\$600,00$ profit. The number of employees and fertilizers required for each hectare are described in Table I. Considering that Manoel can count on 200 employees and 240 tons of fertilizers, how can he maximize his profit?

TABLE I
PROBLEM DATA

	Coffee	Cacoa
Employees	6	4
Fertilizers	4t	8t

The mathematical model for the problem is shown below.

$$\begin{cases} Max: \quad z = 400x_1 + 600x_2 \\ Subject \ to: \\ \qquad x_1 + x_2 \le 90 \\ \qquad 6x_1 + 4x_2 \le 200 \\ \qquad 4x_1 + 8x_2 \le 240 \\ \qquad x_1 \ge 0, x_2 \ge 0 \end{cases} \qquad (3)$$

In this model we have two positive integer decision variables, x_1 which represents the amount of hectares where it will be planted coffee and x_2 the amount of hectares where the cocoa will be planted. Thus, there is a problem of integer linear programming (ILP). The region of the points $(x_1, x_2) \in R^2$ that simultaneously satisfy all the restrictions of the problem is shown in Figure 1. Any point belonging to this region,

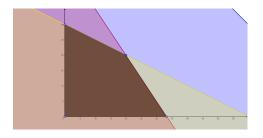satisfies all the restrictions of the problem. We call this the viable region.

Fig. 1. Viable Site Problem Region

Once the viable region is found, we can try to assign values to the objective function and check if the line intersects the viable region. Let $z_0 \ge 0$, $z_0 \in \mathbb{R}$ any value of the line $400x_1 + 600x_2 = z_0$ that intercept the viable region, satisfy the restrictions and have $profit = z_0$. Figure ref objective, shows some of these lines for some values of the objective function. Note that the line $400x_1 + 600x_2 = 10000$ intersects the point (20.20) of the viable region and if we increase the values of z_0 the respective lines will no longer intercept the viable region, this point being the profit point maximum. In fact, an important result for linear optimization problems shows that the viable region is a convex polygon and its optimal solution occurs at one of the vertices of this polygon Bazaraa [1].
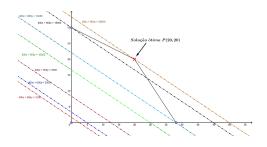

Fig. 2. Objective Function Lines

USE OF GEOGEBRA IN HIGH SCHOOL

Some works found in the literature use GeoGebra as a computational tool, but a small number use it in co-relation with linear programming problems in association with high school mathematical content. Ribas [9] proposed to approach the PL concepts with graphic constructions through GeoGebra. He also used the Excell spreadsheet to make a comparison with GeoGebra. Zachi [11] used GeoGebra to solve economics and business management problems. Pinheiro et al [8] addresses the problems of linear programming using GeoGebra resources to relate practical problems of everyday life to mathematical content learned in class. The authors highlight the valuable gain for students of computer knowledge. Martins [7] reports the need to rethink the curriculum, proposing strategies and alternatives that connect mathematics to everyday life and answer students' questions. Several workshops involving GeoGebra were used in their research activities at the Pastor Dohms High School Center, located in Porto Alegre, totaling

20 hours of classes in 8 meetings. The proposal to relate linear programming to high school mathematical content that we present in this work can be applied in groups or individually and uses GeoGebra to facilitate the study and understanding of the graphics, interpretation of systems of equations and inequalities, as well as visualization of plane regions. GeoGebra is a free software available for computer, notebook, tablet and smartphone, enabling access for all students. Before starting to solve problems the teacher must present GeoGebra to your students, so they can learn to handle it. The teacher must then propose an LP problem to students who must, for a while, reflect, discuss and propose solutions, based on the information provided in the statement. The teacher should not interfere in this stage of the process, as it is necessary for students to propose intuitive resolutions, using previous knowledge, organizing their ideas, raising hypotheses, making tests and discussing possible solutions. After this stage, if no student has done so, the teacher can propose to students to convert the data from the proposed problem from natural language to mathematical language. Thus, it is expected that some linear equations and inequalities are found. For this, the teacher can conduct the work more directly, guiding students, asking them, gradually leading them to build the PL problem as discussed previously. With a LP problem as shown in(5), the teacher can teach Geogebra to his students from the following steps:
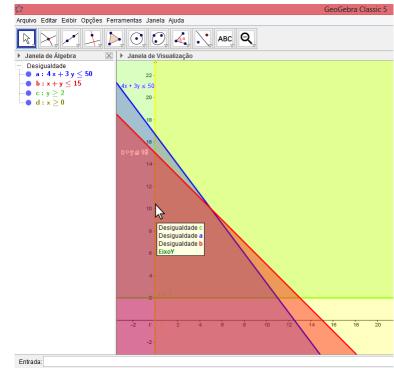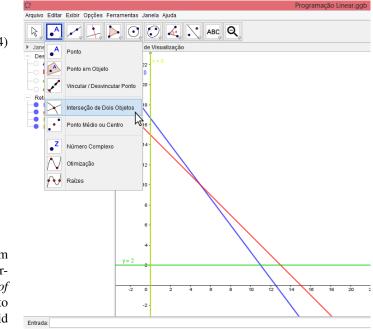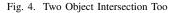

Fig. 3. Intersection of inequalities

$$
\begin{cases}
Max: \quad z = 5x + 4y \\
subject\ to: \\
\qquad 4x + 3y \leq 50 \\
\qquad x + y \leq 15 \\
\qquad x \geq 0, y \geq 2
\end{cases}
\tag{4}
$$

1-Insert in Geogebra all the inequalities of the problem and visualize the intersection (Figure 6). **2**- Insert the corresponding equations in GeoGebra. **3**- Use the *Intersection of Two Objects* tool available on 2^a menu position (Figure 7), to mark the points of intersection of the problem lines. **4** - Build the polygon, using the *Polygon* tool, found in the 5^a menu position, as (Figure 8)

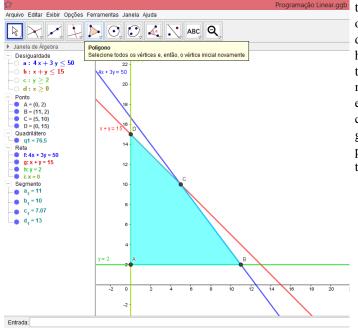Fig. 4. Two Object Intersection Too

Fig. 5. Viable region polygon

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to propose intuitive resolutions, using previous knowledge, organizing their ideas, raising hypotheses, making tests and discussing possible solutions. After this stage, if no student has done so, the teacher can propose to students to convert the data from the proposed problem from natural language to mathematical language. Thus, it is expected that some linear equations and inequalities are found. For this, the teacher can conduct the work more directly, guiding students, asking them, gradually leading them to build the PL problem as discussed previously. With a LP problem as shown in(5), the teacher can teach Geogebra to his students from the following steps:

$$\begin{cases} Max: & z = 5x + 4y \\ subject\ to: & \\ & 4x + 3y \le 50 \\ & x + y \le 15 \\ & x \ge 0, y \ge 2 \end{cases} \quad (5)$$

1-Insert in Geogebra all the inequalities of the problem and visualize the intersection (Figure 6). **2**- Insert the corresponding equations in GeoGebra. **3**- Use the *Intersection of Two Objects* tool available on 2^a menu position (Figure 7), to mark the points of intersection of the problem lines. **4** - Build the polygon, using the *Polygon* tool, found in the 5^a menu position, as (Figure 8)
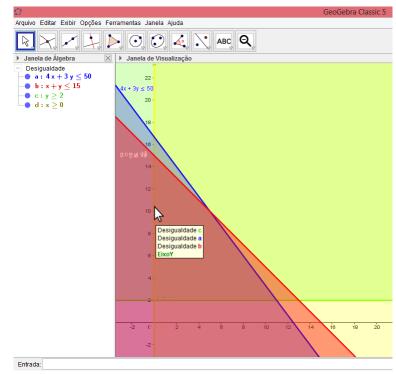

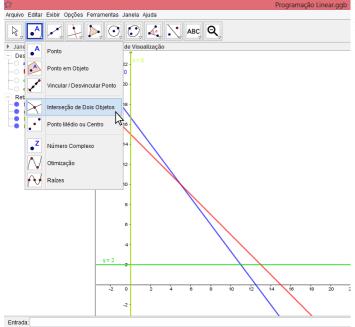Fig. 6. Intersection of inequalities
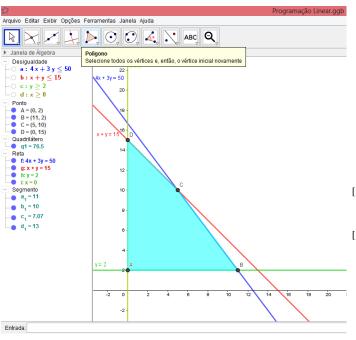
Fig. 7.   Two Object Intersection Too

Fig. 8.   Viable region polygon

Conclusion

In this work, we present an approach to high school mathematical content through GeoGebra. With the introduction of linear programming problems, the teacher will be able to teach his students polygons, angles, sides, vertices, lines, planes, linear equations and inequalities, systems of linear equations and inequalities. the use of the GeoGebra software is an additional motivation for students and will contribute to a better learning of the mentioned contents. Linear programming problems have the advantage of being able to present students with practical day-to-day situations, which can be used to answer some questions from students about the need to study or not a certain mathematical content. For future work it is intended to extend the ideas discussed here to contents of Spatial Geometry.

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