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Mathematical Modeling of the Effects of Optimized Drainage Systems on Mosquito Dynamics in Prevention of Malaria

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Abstract: Malaria continues to pose a major public health threat especially in tropical regions like Nigeria where inadequate drainage systems create stagnant water that fosters mosquito breeding. While conventional interventions such as insecticide-treated nets, indoor spraying, and antimalarial drugs remain essential, environmental management has been comparatively underexplored. This study presents a six-compartment mathematical model that incorporates human and mosquito population dynamics alongside open and closed drainage systems to evaluate their role in controlling malaria transmission. The model integrates susceptible, infected, and recovered classes in both populations and introduces drainage effectiveness parameters (κ_O and κ_C) to simulate the reduction in mosquito habitats. Analytical findings show that when $\kappa_O + \kappa_C > 1$, the disease-free equilibrium becomes locally asymptotically stable which effectively halting endemic transmission. The basic reproduction number (R_0) is derived and results confirm that optimized drainage can bring R_0 below the critical threshold of 1 which is a necessary condition for disease elimination. An optimization framework using Lagrangian techniques further demonstrates that an equal allocation of drainage resources ($\kappa_O D_O = \kappa_C D_C = c/2$) yields the most efficient reduction in mosquito populations. Sensitivity analyses highlight that higher mosquito reproduction (ρ_m) and contact rates (β_2) exacerbate disease persistence there by reinforcing the need for an integrative approaches. This study underscores the potential of drainage optimization as a sustainable, evidence-based addition to malaria control strategies. It offers actionable recommendations for policymakers to integrate environmental management into public health frameworks aimed at achieving malaria elimination by 2030.

Keywords: Mathematical Modeling of Malaria; Mosquito Population Dynamics; Drainage System Optimization; Environmental Vector Control.

Classification MSC: 92D30; 93A30; 49J15.

1 Introduction

Malaria remains one of the most significant public health challenges worldwide, particularly in regions with tropical climate change and urbanization increasing the frequency of stagnant water. Efforts to combat this mosquito-borne disease have historically focused on pharmaceutical interventions and mosquito control measures. However, an often overlooked but highly effective strategy lies in the optimization of drainage systems to manage mosquito breeding habitats. In this study, malaria control strategies will be extend beyond pharmaceutical and chemical interventions to an optimization of drainage systems by disrupting the mosquito life cycle at the source such strategies offer a practical sustainable path toward reducing malaria transmission and ultimately achieving elimination goals. We will explore the mathematical modeling of how optimized drainage systems specifically open and closed drainage systems will impact the population dynamics of mosquitoes and the subsequent transmission of malaria. By examining the impact of drainage

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optimization on mosquito breeding sites and their life cycle, we aim to highlight their potential in reducing mosquito populations and thereby decreasing the incidence of malaria.

Malaria continues to rank among the top five causes of death in low-income regions with the burden falling disproportionately on sub-Saharan Africa. As of late 2024, only a select few African nations such as Algeria, Cabo Verde, Mauritius, and Egypt have achieved malaria-free certification from World Health Organization (2020) highlighting the persistent challenges in malaria elimination. While much of the focus historically has been on pharmaceutical interventions and mosquito control through insecticide-treated nets and indoor spraying, one critical yet frequently neglected factor is environmental management, particularly the role of drainage systems in controlling mosquito breeding. Inadequate drainage leads to stagnant water which provides an ideal habitats for Anopheles mosquitoes which is the principal vectors of malaria. Female adult Anopheline mosquitoes transmit malaria parasitic from infected humans to susceptible humans. After they mate, female adult mosquitoes bite humans (and other animals) and take a blood meal to obtain the protein necessary for egg development and egg laying. Mosquitoes acquire the parasite (Plasmodium) infection by taking blood meals from an infected human. After the parasites complete the development period within the mosquito, then they can subsequently pass the disease to a susceptible human. With urbanization and climate change further exacerbating these life cycle, the role of optimized drainage systems in malaria prevention is increasingly significant.

Nigeria remains a major hotspot for malaria contributing 23% of global cases and 27% of malaria related deaths in 2020 according to African Leaders Malaria Alliance (2024) and World Health Organization (2020) data. Malaria's impact extends beyond health imposing severe socio-economic costs on affected communities, the correlation between poorly managed drainage systems and increased mosquito breeding underscoring the need for sustainable infrastructural approaches alongside medical interventions. Recognizing these realities, the Nigerian government through the National Malaria Elimination Programme (NMEP) has launched a strategic data-driven plan the National Malaria Strategic Plan aimed at reducing malaria morbidity and mortality. This approach prioritizes the allocation of resources based on local needs and targets interventions at the Local Government Area (LGA) level in line with the WHO's High Burden to High Impact strategy.

Mathematical modeling now plays an essential role in informing these strategies offering insights into the transmission dynamics of malaria and the probable effects of various interventions, such models enable policymakers to simulate several scenarios in identify critical points of intervention and optimize control measures. A pivotal aspect of these strategies is the environmental management, specifically the optimization of drainage systems with Open drains which is commonly found in both urban and rural Nigeria often accumulate stagnant water creating optimal breeding conditions for malaria vectors while on the converse, a well-designed and regularly maintained drainage infrastructure can significantly reduce vector populations by limiting breeding sites.

Recent years have seen growing advocacy for integrating environmental management into malaria control efforts especially those targeting water management and drainage optimization are recognized as sustainable complements to traditional interventions. Effective drainage requires a combination of sound engineering, ongoing maintenance, and community engagement to ensure that systems function as intended and do not become mosquito habitats. Numerous researchers, including Castro et al. (2023) and Jakada et al. (2023) have examined the impact of community-based environmental management on malaria control. Their work demonstrates that interventions targeting the environmental determinants of vector breeding can yield substantial, lasting reductions in disease transmission. Drawing inspiration from the African Leaders Malaria Alliance (2024)'s report which urges accelerated efforts toward malaria elimination by 2030, this study sets out to construct a mathematical model specifically examining the impact of optimized drainage systems on mosquito population dynamics and malaria transmission. Both open and closed drainage interventions are incorporated with the aim of evaluating their respective effectiveness in reducing mosquito populations and lowering malaria incidence. The central objective is to develop a model that integrates both open and closed drainage strategies, allowing for a clear comparison of their influence on malaria transmission

rates. The research will determine the existence and stability of model equilibria, supported by numerical simulations to validate the analytical findings. Sensitivity analyses will also be conducted to highlight the most influential parameters, emphasizing the importance of drainage interventions in comprehensive malaria prevention strategies.

In an ongoing effort to combat malaria, it is critical to adopt innovative approaches that address both biological and environmental determinants of disease transmission. Previous research, such as Jakada et al. (2023) and Mokuolua et al. (2017), has shed light on the relationship between malaria prevalence and environmental factors, primarily through statistical analysis. While these statistical approaches yield valuable data, they may not fully capture the dynamic interplay between mosquito populations and their breeding environments particularly with respect to open drainage systems. To address this gap, this article extend the approach of Mokuolua et al. (2017) by implementing a compartmental model that allows for a more nuanced representation of malaria transmission dynamics. Compartmental models are widely utilized in epidemiology to simulate the movement of individuals between states (e.g., susceptible, infected, recovered) and can incorporate the effects of interventions over time. By integrating this framework, the study will provide a deeper analysis of how optimized drainage systems especially open drainage affect mosquito habitats and malaria transmission rates.

The Open drainage systems which are common in many malaria endemic regions play a significant role in fostering environments conducive to mosquito breeding. By focusing on these infrastructures, this research offers a targeted examination of how environmental interventions shape malaria dynamics. The anticipated outcome is to inform public health policies and infrastructure planning which ultimately contributed to a more effective strategies for malaria control. Through this enhanced modeling approach, the study seeks to bridge the gap between statistical and dynamic modeling, offering a comprehensive perspective that underscores the critical role of environmental management in malaria prevention. The critical role of mathematical modeling in understanding malaria transmission and evaluating control measures as emphasized by Collins and Duffy (2023) includes addressing challenges posed by drug-resistant malaria strains, seasonal variations, and the interplay between resistance and control strategies like mosquito nets and improved treatments. Techniques such as sensitivity analysis, optimal control models, and data integration have advanced knowledge significantly, yet gaps remain, including the need for comprehensive models specific to Nigeria that integrate drug resistance, control measures, and real data, alongside long-term predictions addressing socio-economic and environmental factors.

In alignment with these findings, Edossa and Koya (2020) utilized a mathematical model to evaluate the efficacy of control strategies such as insecticide-treated bed nets (ITNs), indoor residual spray (IRS), environmental management, and antimalarial drug treatments. Their results reveal the effectiveness of ITNs as the most impactful intervention particularly when combined with other measures. Futhermore, the numerical simulations demonstrated that integrating all four strategies significantly reduces malaria transmission, however, their study also highlights gaps in long-term sustainability, community acceptance, socio-economic impacts, and insecticide resistance. Together with these findings reinforce the importance of developing integrated region-specific approaches to malaria control that address both technical and societal challenges, ultimately enhancing public health outcomes in regions like Nigeria.

Complementing these insights, the study by Castro et al. (2023) on community-based environmental management in Dar es Salaam demonstrated the effectiveness of community sensitization and drain maintenance in reducing malaria transmission, with intervention areas showing significantly lower Odds Ratio (OR = 0.12) of infection. However, resource limitations and inconsistent commitment hindered long-term success. Together, these studies underscore the need for integrated, sustainable, and community-driven approaches to malaria control, emphasizing the importance of combining mathematical modeling with practical interventions and addressing societal challenges to enhance public health outcomes. Similarly, the study by Dauda et al. (2023), Jakada et al. (2023), and Koissi Savi (2023) shows the importance of sanitation practices in reducing mosquito breeding. The study revealed that poor sanitation practices

influenced by socio-demographic factors contribute significantly to mosquito proliferation recommending the integration of sanitation into malaria programs and improved waste management. However, gaps such as limited geographic scope, insufficient sample diversity, and lack of long-term assessments highlight the need for broader studies and enhanced community engagement. Collectively, these findings emphasize the necessity of combining mathematical modeling with practical, community-driven interventions and policy measures to tackle the multifaceted challenges of malaria control effectively.

In support of these perspectives, the study by Ojo (2020) employs a nonlinear deterministic compartmental model to understand malaria transmission dynamics Ozodiegwu et al. (2023). The research analyzes the impact of bed nets, treatments, and insecticides using stability theory, optimal control theory, and numerical simulations. Key findings from the study include the evaluation of both single and combined control strategies, with graphical representations of their effects. However, the study also identifies significant gaps, such as the limited scope of control strategies, insufficient adaptation to local contexts, and the lack of long-term sustainability analysis. Techniques utilized in the study—mathematical modeling, stability theory, and optimal control formulations—provide valuable insights into malaria control strategies and public health interventions. These insights further contribute to the understanding of the complex malaria control landscape, emphasizing the need for broader, more context-specific models and long-term strategies, particularly in regions like Nigeria.

Additionally Evans et al. (2020)present a deterministic model focused on mosquito population dynamics. The model specifically examines insecticide-sensitive and insecticide-resistant mosquito types establishing conditions for the stability of four equilibrium points validated through numerical simulations. The study highlights the pivotal role of female Anopheline mosquitoes in malaria transmission, an insight critical to developing effective control strategies. However, it also identifies gaps including a limited understanding of the impacts of insecticide resistance, the absence of spatial and environmental factors, and insufficient insights into mosquito-human infection interactions. The techniques used in the study equilibrium analysis, differential equations, and simulations offer valuable perspectives on enhancing mosquito control strategies and reducing disease transmission.

Building on the studies of Koissi Savi (2023) which highlighted the multifaceted challenges of malaria control emphasizing its complex nonlinear dynamics. They identifies key gaps including limited understanding of the disease's complexity, the failure of past interventions to address resistance issues, the need for multidisciplinary approaches, and significant information asymmetry that hinders effective policy-making. Koissi Savi (2023) adopts a systems approach to analyze the interactions in malaria transmission using mathematical and simulation models to predict disease dynamics and providing didactic material to improve public understanding. By integrating interdisciplinary insights from public health, economics, and environmental science. Their work aims to inform more effective malaria control strategies and future policy development. In line with these mathematical modeling and intervention strategies Strugarek et al. (2023) explores additional vector control methods particularly the Sterile Insect Technique (SIT) and Incompatible Insect Technique (IIT) to combat mosquito-borne diseases like dengue and Zika. The study uses mathematical models to evaluate the effectiveness of SIT and IIT, analyzing different release strategies and identifying conditions for successful mosquito population control. However, the research highlights gaps such as the need for field verification, accurate parameter identification, and practical implementation strategies. By employing the mass release of sterilized or Wolbachia-infected male mosquitoes, the study underscores the importance of tailored release strategies and further research to optimize these vector control methods. Collectively, these studies point to the necessity of combining mathematical modeling with practical, interdisciplinary, and community-driven interventions to enhance mosquito control and malaria prevention efforts globally.

Finally, Montoya and Romero-Leiton (2019) present two mathematical models focused on malaria transmission, with particular attention to resistance and population movement. The single patch model analyzes interactions within a single area, while the two patch model explores population movement

between regions. The study identifies several gaps, including limited real-world application, insufficient exploration of resistance mechanisms, and the lack of consideration for human behavior and socio-economic factors. Techniques such as ordinary differential equations (ODEs), stability analysis, bifurcation analysis, sensitivity analysis, optimal control theory, and numerical experiments provide valuable insights into malaria dynamics, aiming to enhance control strategies and improve disease management. Recently, Ozodiegwu et al. (2023) utilizes agent-based modeling to simulate malaria morbidity and mortality across Nigeria's 774 local government areas (LGAs). Their analysis of four intervention strategies, including business-as-usual and resource-prioritized plans, underscores the need for improved subnational data collection systems and a better understanding of resource optimization. The study applies agent-based modeling to simulate transmission dynamics, epidemiological archetype clustering to tailor interventions and scenario analysis to evaluate the impacts of different strategies. Addressing these gaps with advanced modeling techniques can significantly enhance the effectiveness of malaria interventions in Nigeria.

These collective studies underscore the critical role of mathematical models in enhancing malaria control strategies, while the studies mentioned above primarily focus on various aspects of malaria transmission and mosquito control, including insecticide resistance, ITNs, IRS, and other vector control methods. However, none of the studies explicitly addressed the optimization of drainage systems as a control method for mosquito dynamics. That being said, some of the studies, such as Castro et al. (2023)'s work on community-based environmental management in Dar es Salaam, touch upon the importance of environmental management, including sanitation and drain maintenance, as part of a broader strategy to reduce malaria transmission. Castro et al. (2023)'s research indicates that community-driven environmental management, Similarly, while the study by Jakada et al. (2023) emphasizes sanitation practices to reduce mosquito breeding.

By extending Mokuolua et al. (2017) research to explore the impact of proper channeling of drainage in malaria control with a deeper focus on optimizing drainage systems, particularly the proper channeling of water in urban and semi-urban environments, could further enhance malaria control efforts by minimizing breeding sites for mosquitoes. This study is motivated by the report of African Leaders Malaria Alliance African Leaders Malaria Alliance, 2024 to ultimately accelerating Africa's progress toward malaria elimination by 2030. The focus of this study is to develop a mathematical model that explores the impact of optimized drainage systems on mosquito population dynamics and malaria transmission by incorporating both open and closed drainage interventions into the model. We aim to evaluate their effectiveness in controlling mosquito populations and reducing malaria incidence.

2 Model Formulation

We develop a six-compartment model to describe the dynamics of human and mosquito populations. The model incorporates systems such as open and closed drainage to analyze their effects of mosquitoes breeding and disease transmission.

1. **Population Dynamics:** The human population is assumed constant with no migration, and homogeneous mixing is considered between humans and mosquitoes.

2. **Environmental and Drainage Factors:** Mosquito breeding is directly influenced by drainage efficiency, but the model does not explicitly capture complex hydrological processes like rainfall, topography, or infrastructure design.

3. **Disease and Immunity Assumptions:** The model assumes no resistance to drugs or insecticides and that recovered individuals gain temporary immunity.

4. **Simplified Mosquito Life Cycle:** The mosquito life stages are aggregated into compartments for computational simplicity.

5. **Exclusion of Human Behavior:** The model does not account for human behavioral responses to malaria interventions such as net usage or community-based environmental practices.

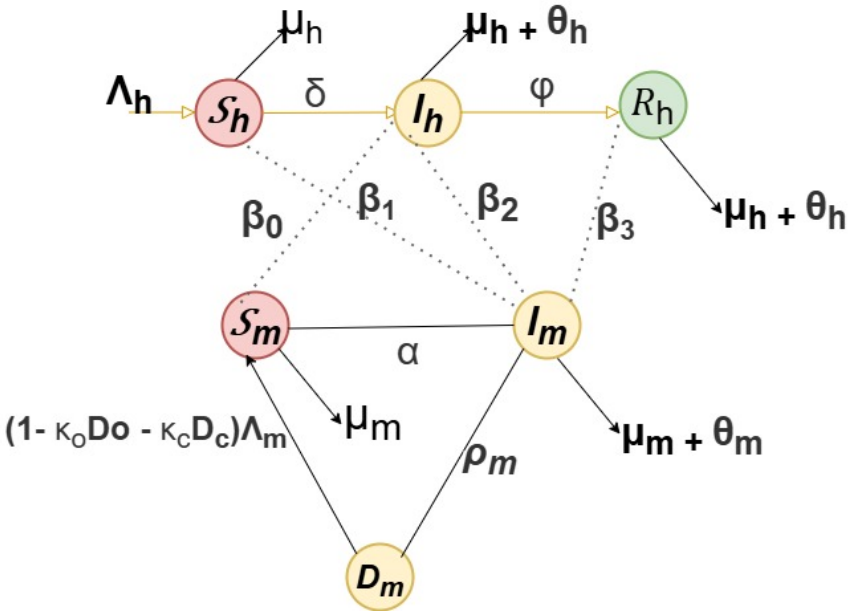


Fig. 1. This is a sample image.

$$
\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \frac{\lambda_m S_h}{N_h} - (\mu_h + \beta_1) S_h \\
\frac{dI_h}{dt} &= \frac{\lambda_h S_h}{N_h} + \beta_0 S_m - (\mu_h + \delta_h + \sigma + \beta_2)\, I_h \\
\frac{dR_h}{dt} &= \beta_3 I_m - \sigma I_h - (\mu_h + \delta_h) R_h \\
\frac{dS_m}{dt} &= (1 - \kappa_O D_O - \kappa_C D_C)\, \Lambda_m D_m + \frac{\lambda_m I_h}{N_m} - (\mu_m + \beta_0) S_m \\
\frac{dI_m}{dt} &= \beta_2 I_h - (\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \frac{\lambda_m}{N_m}) I_m \\
\frac{dD_m}{dt} &= \rho_m I_m - (1 - \kappa_O D_O - \kappa_C D_C)\, \Lambda_m D_m \\
\frac{dN_h}{dt} &= \frac{dS_h}{dt} + \frac{dI_h}{dt} + \frac{dR_h}{dt} \\
\frac{dN_m}{dt} &= \frac{d(S_m + I_m + D_M)}{dt}.
\end{aligned}
\tag{1}
$$

The total population derivatives are computed as:

$$
\begin{aligned}
\frac{dN_h}{dt} &= \Lambda_h - \mu_h N_h - \beta_1 S_h + \frac{\lambda_h S_h}{N_h} + \beta_0 S_m - (\delta_h + \sigma + \beta_2) I_h + \beta_3 I_m - \sigma I_h - \delta_h R_h, \\
\frac{dN_m}{dt} &= \frac{\lambda_m I_h}{N_m} - (\mu_m + \beta_0) S_m + \beta_2 I_h - (\beta_1 + \beta_3 + \mu_m + \delta_m + \frac{\lambda_m}{N_m}) I_m.
\end{aligned}
$$

Variables		Variables (cont.)	
S_h	Susceptible humans	S_m	Susceptible mosquitoes
I_h	Infected humans	I_m	Infected mosquitoes
R_h	Recovered humans	D_m	Mosquito drainage level
Parameters		**Parameters (cont.)**	
Λ_h	Recruitment rate of humans	μ_h, μ_m	Natural death rate of humans, mosquitoes respectively
δ_h, δ_m	Disease-induced death rate of humans, mosquitoes respectively	σ	Recovery rate of humans
β_0	Contact rate of susceptible mosquitoes and infected humans	β_1	Contact rate of susceptible humans and infected mosquitoes
β_2	Contact rate of infective mosquitoes and infective humans	β_3	Contact rate of infective mosquitoes and recovered humans
λ_h, λ_m	Rate of disease transmission in humans, mosquitoes respectively	ρ_m	Reproduction rate of mosquitoes
$(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m$	Modified birth rate of mosquitoes due to drainage	$\kappa_O D_O$	Effectiveness of open drainage on mosquito population
$\kappa_C D_C$	Effectiveness of closed drainage on mosquito population		

Table 1. Description of variables and parameters

2.1 Basic Properties of the Model Equation

Model Description

Consider the following malaria transmission model:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \frac{\lambda_m S_h}{N_h} - (\mu_h + \beta_1)S_h, \\
\frac{dI_h}{dt} &= \frac{\lambda_h S_h}{N_h} + \beta_0 S_m - (\mu_h + \delta_h + \sigma + \beta_2)I_h, \\
\frac{dR_h}{dt} &= \beta_3 I_m - \sigma I_h - (\mu_h + \delta_h)R_h, \\
\frac{dS_m}{dt} &= (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m D_m + \frac{\lambda_m I_h}{N_m} - (\mu_m + \beta_0)S_m, \\
\frac{dI_m}{dt} &= \beta_2 I_h - \left(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \frac{\lambda_m}{N_m}\right) I_m, \\
\frac{dD_m}{dt} &= \rho_m I_m - (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m D_m,
\end{aligned}$$

where $N_h = S_h + I_h + R_h$ and $N_m = S_m + I_m + D_m$ represent the total human and mosquito populations, respectively.

Non-Negativity of Solutions

Theorem 1. *If the initial conditions satisfy $S_h(0), I_h(0), R_h(0), S_m(0), I_m(0), D_m(0) \geq 0$, then the solutions $S_h(t), I_h(t), R_h(t), S_m(t), I_m(t), D_m(t)$ remain non-negative for all $t > 0$.*

Proof. We analyze each compartment individually.

Non-Negativity of $S_h(t)$

From the first equation:

$$\frac{dS_h}{dt} = \Lambda_h - \frac{\lambda_m S_h}{N_h} - (\mu_h + \beta_1)S_h \geq \Lambda_h - \left(\frac{\lambda_m}{N_h} + \mu_h + \beta_1\right) S_h.$$

Let $\phi(t) = \frac{\lambda_m(t)}{N_h(t)} + \mu_h + \beta_1$. Then:

$$\frac{dS_h}{dt} + \phi(t)S_h \geq \Lambda_h.$$

Using the integrating factor $\mu(t) = \exp\left(\int_0^t \phi(s)ds\right)$, we obtain:

$$\frac{d}{dt}\left(S_h(t)\mu(t)\right) \geq \Lambda_h\mu(t).$$

Integrating from 0 to t:

$$S_h(t)\mu(t) - S_h(0) \geq \Lambda_h \int_0^t \mu(s)ds,$$

which implies:

$$S_h(t) \geq S_h(0)\exp\left(-\int_0^t \phi(s)ds\right) + \Lambda_h \exp\left(-\int_0^t \phi(s)ds\right)\int_0^t \exp\left(\int_0^s \phi(u)du\right)ds > 0.$$

Thus, $S_h(t) > 0$ for all $t > 0$.
Similarly, it can be shown that From the second equation:

$$\frac{dI_h}{dt} = \frac{\lambda_h S_h}{N_h} + \beta_0 S_m - (\mu_h + \delta_h + \sigma + \beta_2)I_h \geq -(\mu_h + \delta_h + \sigma + \beta_2)I_h.$$

This yields:

$$\frac{dI_h}{dt} + \eta(t)I_h \geq 0, \quad \text{where } \eta(t) = \mu_h + \delta_h + \sigma + \beta_2.$$

Thus:

$$I_h(t) \geq I_h(0)e^{-\int_0^t \eta(s)ds} \geq 0.$$

From the third equation:

$$\frac{dR_h}{dt} = \beta_3 I_m - \sigma I_h - (\mu_h + \delta_h)R_h \geq -(\mu_h + \delta_h)R_h.$$

This gives:

$$R_h(t) \geq R_h(0)e^{-(\mu_h+\delta_h)t} \geq 0.$$

Non-Negativity of $S_m(t), I_m(t), D_m(t)$ follows similar arguments apply to the mosquito compartments:

- For $S_m(t)$, the equation has non-negative inflow terms and linear decay, ensuring $S_m(t) \geq 0$.
- For $I_m(t)$, the inflow $\beta_2 I_h$ is non-negative, and the decay is linear, so $I_m(t) \geq 0$.
- For $D_m(t)$, the inflow $\rho_m I_m$ is non-negative, and the decay term is bounded, ensuring $D_m(t) \geq 0$.

□

We prove that the region Ω is invariant for the system of differential equations. The invariant region Ω is defined as:

$$\Omega = \left\{(S_h, I_h, R_h, S_m, I_m, D_m) \in \mathbb{R}_+^6 \,\middle|\, N_h \leq \frac{\Lambda_h}{\mu_h},\ N_m \leq \frac{\Lambda_m}{\mu_m}\right\},$$

Theorem 2. *The total populations $N_h(t)$ and $N_m(t)$ are bounded for all $t > 0$.*

Proof. Summing the human equations:

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h - \delta_h(I_h + R_h) \leq \Lambda_h - \mu_h N_h.$$

This implies:

$$N_h(t) \leq N_h(0)e^{-\mu_h t} + \frac{\Lambda_h}{\mu_h}\left(1 - e^{-\mu_h t}\right) \leq \max\left(N_h(0), \frac{\Lambda_h}{\mu_h}\right).$$

Summing the mosquito equations:

$$\frac{dN_m}{dt} = \Lambda_m D_m(1 - \kappa_O D_O - \kappa_C D_C) - \mu_m N_m - \delta_m I_m \leq \Lambda_m - \mu_m N_m.$$

Thus:

$$N_m(t) \leq N_m(0)e^{-\mu_m t} + \frac{\Lambda_m}{\mu_m}\left(1 - e^{-\mu_m t}\right) \leq \max\left(N_m(0), \frac{\Lambda_m}{\mu_m}\right).$$

□

Positively Invariant Region

Define the region:

$$\mathcal{D} = \left\{(S_h, I_h, R_h, S_m, I_m, D_m) \in \mathbb{R}_+^6 \,:\, N_h \leq \frac{\Lambda_h}{\mu_h},\, N_m \leq \frac{\Lambda_m}{\mu_m}\right\}.$$

Theorem 3. *The region $\mathcal{D}$ is positively invariant under the model dynamics.*

Proof. From the boundedness results, any solution starting in $\mathcal{D}$ remains in $\mathcal{D}$ for all $t > 0$. Non-negativity ensures that $\mathcal{D}$ is invariant. □

2.1.1 Invariant Region of the Model

We prove that $N_h \leq \frac{\Lambda_h}{\mu_h}$ and $N_m \leq \frac{\Lambda_m}{\mu_m}$ for all $t \geq 0$.

Proof for N_h:

From the equation for $\frac{dN_h}{dt}$:

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h + \left(\frac{\lambda_h - \lambda_m}{N_h}\right) S_h + \beta_0 S_m - (\delta_h + 2\sigma + \beta_2) I_h + \beta_3 I_m - \delta_h R_h.$$

At steady state ($\frac{dN_h}{dt} = 0$):

$$\Lambda_h - \mu_h N_h^* = 0 \implies N_h^* = \frac{\Lambda_h}{\mu_h}.$$

To show boundedness, consider:

$$\frac{dN_h}{dt} \leq \Lambda_h - \mu_h N_h.$$

This is a first-order linear differential inequality. Solving it:

$$N_h(t) \leq \frac{\Lambda_h}{\mu_h} + \left(N_h(0) - \frac{\Lambda_h}{\mu_h}\right) e^{-\mu_h t}.$$

As $t \to \infty$, $N_h(t) \leq \frac{\Lambda_h}{\mu_h}$. Thus, N_h is bounded above by $\frac{\Lambda_h}{\mu_h}$.

Proof for N_m***:***

From the equation for $\frac{dN_m}{dt}$:

$$\frac{dN_m}{dt} = \frac{\lambda_m I_h}{N_m} - (\mu_m + \beta_0)S_m + \beta_2 I_h - \left(\beta_1 + \beta_3 + \mu_m + \delta_m + \frac{\lambda_m}{N_m} - \rho_m\right) I_m.$$

At steady state ($\frac{dN_m}{dt} = 0$):

$$\frac{\lambda_m I_h}{N_m^*} - (\mu_m + \beta_0)S_m + \beta_2 I_h - \left(\beta_1 + \beta_3 + \mu_m + \delta_m + \frac{\lambda_m}{N_m^*} - \rho_m\right) I_m = 0.$$

To show boundedness, consider:

$$\frac{dN_m}{dt} \leq \Lambda_m - \mu_m N_m.$$

This is a first-order linear differential inequality. Solving it:

$$N_m(t) \leq \frac{\Lambda_m}{\mu_m} + \left(N_m(0) - \frac{\Lambda_m}{\mu_m}\right) e^{-\mu_m t}.$$

As $t \to \infty$, $N_m(t) \leq \frac{\Lambda_m}{\mu_m}$.
Thus, N_m is bounded above by $\frac{\Lambda_m}{\mu_m}$.

The invariant region Ω is defined as:

$$\Omega = \left\{(S_h, I_h, R_h, S_m, I_m, D_m) \in \mathbb{R}_+^6 \,\middle|\, N_h \leq \frac{\Lambda_h}{\mu_h},\ N_m \leq \frac{\Lambda_m}{\mu_m}\right\}.$$

Theorem 4 (LaSalle's Invariance Principle)**.** *Let $\dot{x} = f(x)$ be a dynamical system where $f(x)$ is continuously differentiable and $D \subseteq \mathbb{R}^n$ is a positively invariant region. If the system has a Lyapunov function $V(x)$ such that:*

$$\dot{V}(x) = \nabla V(x) \cdot f(x) \leq 0, \quad \forall x \in D,$$

then the set $\mathcal{L} = \{x \in D : \dot{V}(x) = 0\}$ is invariant, and the trajectory of the system will tend to an equilibrium point or a limit cycle within $\mathcal{L}$.

Lemma 5. *Let*

$$\mathcal{R} = \left\{(S_h, I_h, R_h, S_m, I_m, D_m) \in \mathbb{R}_+^6 : S_h + I_h + R_h \leq \frac{\Lambda_h}{\mu_h},\ S_m + I_m + D_m \leq \frac{(1 - \kappa_O D_O - \kappa_C D_C)\,\Lambda_m}{\mu_m}\right\}.$$

The trajectories of the system remain within $\mathcal{R}$, as guaranteed by LaSalle's Invariance Principle 4.

Proof. From the Lyapunov function $V(x) = S_h + I_h + R_h + S_m + I_m + D_m$, we observe:

$$\frac{dV}{dt} = -\mu_h(S_h + I_h + R_h - \frac{\Lambda_h}{\mu_h}) - \mu_m \left(S_m + I_m + D_m - \frac{(1 - \kappa_O D_O - \kappa_C D_C)\,\Lambda_m}{\mu_m} \right).$$

Since $\mu_h, \mu_m > 0$, it follows that $\frac{dV}{dt} \leq 0$ whenever the populations are within $\mathcal{R}$. This ensures that the total population remains bounded and the solutions of the system are non-increasing within the region.

By LaSalle's Invariance Principle, the trajectories will remain within $\mathcal{R}$ and eventually converge to an equilibrium point where $\frac{dV}{dt} = 0$. This equilibrium corresponds to the steady-state population levels $\frac{\Lambda_h}{\mu_h}$ for humans and $\frac{(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m}{\mu_m}$ for mosquitoes.
Hence, the invariant region $\mathcal{R}$ is biologically meaningful, capturing the feasible range of the system dynamics. □

2.2 Disease-Free Equilibrium

We analyze the disease-free equilibrium (DFE) E_0 of the system of differential equations and prove its stability.

In the disease-free equilibrium, we assume:

$$I_h = 0, \quad I_m = 0, \quad D_m = 0.$$

at equilibrium

$$\frac{dS_h}{dt} = \quad \frac{dI_h}{dt} = \quad \frac{dR_h}{dt} = \quad \frac{dS_m}{dt} = \quad \frac{dI_m}{dt} = \quad \frac{dD_m}{dt} = \quad \frac{dN_h}{dt} = \quad \frac{dN_m}{dt} = 0$$

$$\Lambda_h - S_h \frac{\lambda_m}{N_h} - (\mu_h + \beta_1) S_h = 0.$$

Thus from first equation in (1):

$$\Lambda_h - (\mu_h + \beta_1) S_h = 0 \implies S_h = \frac{\Lambda_h}{\mu_h + \beta_1}.$$

From the second equation in (1):

$$\frac{\lambda_h S_h}{N_h} + \beta_0 S_m - (\mu_h + \delta_h + \sigma + \beta_2) I_h = 0.$$

$$I_h = \frac{\frac{\lambda_h S_h}{N_h} + \beta_0 S_m}{\mu_h + \delta_h + \sigma + \beta_2}$$

$$I_h = \frac{\lambda_h \left(\frac{\Lambda_h}{\mu_h + \beta_1} \right) + \beta_0 S_m}{\mu_h + \delta_h + \sigma + \beta_2}$$

from third equation in (1):

$$\beta_3 I_m - \sigma I_h - (\mu_h + \delta_h) R_h = 0.$$

$$R_h = \frac{\beta_3 I_m - \sigma I_h}{\mu_h + \delta_h}$$

from the fourth equation in (1):

$$(1-\kappa_O D_O-\kappa_C D_C)\Lambda_m D_m+\frac{\lambda_m I_h}{N_m}-(\mu_m+\beta_0)S_m=0.$$

$$S_m=\frac{(1-\kappa_O D_O-\kappa_C D_C)\Lambda_m D_m+\frac{\lambda_m I_h}{N_m}}{\mu_m+\beta_0}$$

from the fifth equation in (1):

$$\beta_2 I_h-(\beta_1+\beta_3+\rho_m+\mu_m+\delta_m+\frac{\lambda_m}{N_m})I_m=0.$$

$$I_m=\frac{\beta_2 I_h}{\beta_1+\beta_3+\rho_m+\mu_m+\delta_m+\frac{\lambda_m}{N_m}}$$

from the sixth equation in (1):

$$\rho_m I_m-(1-\kappa_O D_O-\kappa_C D_C)\Lambda_m D_m=0.$$

$$D_m=\frac{\rho_m I_m}{(1-\kappa_O D_O-\kappa_C D_C)\Lambda_m}$$

From the above analysis, the disease-free equilibrium E_0 is:

$$E_0=(S_h^*,I_h^*,R_h^*,S_m^*,I_m^*,D_m^*)=\left(\frac{\Lambda_h}{\mu_h+\beta_1},0,0,0,0,0\right).$$

2.3 Endemic Equilibrium Conditions

From all the equilibrium values, we obtain the endemic equilibrium condition for the model as

$$D=(K_3)(K_1)[K_2(\mu_m+\beta_0)-\beta_0(\tfrac{\rho_m\beta_2+\lambda_m K_1}{K_1})]$$

$$\begin{aligned}K_1&=\beta_1+\beta_3+\rho_m+\mu_m+\delta_m+\lambda_m\\K_2&=\mu_h+\delta_h+\sigma+\beta_2\\K_3&=\lambda_m+\mu_h+\beta_1\end{aligned}$$

$$
\begin{aligned}
S_h^* &= \frac{\Lambda_h}{\lambda_m + \mu_h + \beta_1}, \\
S_m^* &= \frac{(\rho_m\beta_2 + \lambda_m K_1)\lambda_h\Lambda_h}{K_1K_2(\mu_m + \beta_0)(\lambda_m + \mu_h + \beta_1) - (\rho_m\beta_2 + \lambda_m K_1)\beta_0(\lambda_m + \mu_h + \beta_1)}, \\
I_m^* &= \frac{\beta_2\lambda_h\Lambda_h(\mu_m + \beta_0)}{K_3\left[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)\right]}, \\
I_h^* &= \frac{\lambda_h\Lambda_h K_1(\mu_m + \beta_0)}{K_3\left[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)\right]}, \\
D_m^* &= \frac{\rho_m\beta_2\lambda_h(\mu_m + \beta_0)}{(1 - \kappa_O D_O - \kappa_C D_C)\cdot\mathcal{D}}, \\
R_h^* &= \frac{\lambda_h\Lambda_h(\mu_m + \beta_0)(\beta_3\beta_2 - \sigma(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m))}{(\mu_h + \delta_h)(\lambda_m + \mu_h + \beta_1)\left[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)\right]}.
\end{aligned}
\tag{2}
$$

2.4 Basic Reproduction Number R_0

Given the system of differential equations, we identify the infected compartments as:

$$
x = \begin{bmatrix} I_h \\ I_m \\ D_m \end{bmatrix}
$$

We will apply the Next Generation Matrix (NGM) method (van den Driessche and Watmough, 2002) by constructing matrices $\mathcal{F}$ and $\mathcal{V}$ such that:

$$
\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x)
$$

$$
\mathcal{F} = \begin{bmatrix} \frac{\beta_0 I_m S_h^*}{N_h} \\ \beta_2 I_h \\ 0 \end{bmatrix}
$$

$$
\mathcal{V} = \begin{bmatrix} (\mu_h + \delta_h + \sigma + \beta_2)I_h \\ (\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \frac{\lambda_m}{N_m})I_m - \beta_2 I_h \\ -\rho_m I_m + (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m D_m \end{bmatrix}
$$

At the Disease-Free Equilibrium (DFE), we have: $I_h = I_m = D_m = 0$, and thus the Jacobians are evaluated at this point.

Let $S_h^* = \dfrac{\Lambda_h}{\mu_h + \beta_1}$ at DFE. Define the following constants:

$$a = \mu_h + \delta_h + \sigma + \beta_2, \quad b = \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m$$

The Jacobian of $\mathcal{F}$ is:

$$F = \begin{bmatrix} 0 & \dfrac{\beta_0 S_h^*}{N_h} & 0 \\ \beta_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The Jacobian of $\mathcal{V}$ at DFE is:

$$V = \begin{bmatrix} a & 0 & 0 \\ -\beta_2 & b & 0 \\ 0 & -\rho_m & \Lambda_m \end{bmatrix}$$

Since the third row/column does not feed into new infections, we consider the reduced 2x2 system:

$$F_r = \begin{bmatrix} 0 & \dfrac{\beta_0 S_h^*}{N_h} \\ \beta_2 & 0 \end{bmatrix}, \quad V_r = \begin{bmatrix} a & 0 \\ -\beta_2 & b \end{bmatrix}$$

The inverse of V_r is:

$$V_r^{-1} = \frac{1}{ab} \begin{bmatrix} b & 0 \\ \beta_2 & a \end{bmatrix}$$

Now compute the next generation matrix $K = F_r V_r^{-1}$:

$$K = F_r V_r^{-1} = \frac{1}{ab} \begin{bmatrix} 0 & \dfrac{\beta_0 S_h^*}{N_h} \\ \beta_2 & 0 \end{bmatrix} \begin{bmatrix} b & 0 \\ \beta_2 & a \end{bmatrix} = \frac{1}{ab} \begin{bmatrix} \dfrac{\beta_0 \beta_2 S_h^*}{N_h} & \dfrac{\beta_0 a S_h^*}{N_h} \\ b\beta_2 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & x \\ y & 0 \end{bmatrix}, \quad \text{we have eigenvalues } \lambda = \pm\sqrt{xy}$$

So the basic reproduction number is:

$$R_0 = \rho(K) = \sqrt{\frac{\beta_0 \beta_2 S_h^*}{N_h ab}}$$

Since $S_h^* = \dfrac{\Lambda_h}{\mu_h + \beta_1}$ and we consider for a case of small number N_h, the expression for R_0 becomes:

$$R_0 = \sqrt{\frac{\beta_0 \beta_2 \Lambda_h}{(\mu_h + \beta_1)(\mu_h + \delta_h + \sigma + \beta_2)(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m)}}$$

2.5 Stability Analysis

Theorem 6. *(Poincaré-Bendixson Theorem) Let* $\mathbf{X}(t)$ *be a continuous dynamical system in a two-dimensional plane, with solutions that are confined to a bounded region. If the system does not exhibit behavior where trajectories spiral out to infinity or exhibit chaotic dynamics, then there exists a stable equilibrium point such as a fixed point or limit cycle that the system approaches.*

To analyze the stability of E_0, we linearize the system around E_0 and compute the Jacobian matrix. The eigenvalues of the Jacobian matrix determine the stability; If all eigenvalues have negative real parts E_0 is locally asymptotically stable and If any eigenvalue has a positive real part E_0 is unstable.

Jacobian Matrix (J)***:***

The Jacobian matrix (J) at E_0 is computed as:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial S_h} & \frac{\partial f_1}{\partial I_h} & \frac{\partial f_1}{\partial R_h} & \frac{\partial f_1}{\partial S_m} & \frac{\partial f_1}{\partial I_m} & \frac{\partial f_1}{\partial D_m} \\ \frac{\partial f_2}{\partial S_h} & \frac{\partial f_2}{\partial I_h} & \frac{\partial f_2}{\partial R_h} & \frac{\partial f_2}{\partial S_m} & \frac{\partial f_2}{\partial I_m} & \frac{\partial f_2}{\partial D_m} \\ \frac{\partial f_3}{\partial S_h} & \frac{\partial f_3}{\partial I_h} & \frac{\partial f_3}{\partial R_h} & \frac{\partial f_3}{\partial S_m} & \frac{\partial f_3}{\partial I_m} & \frac{\partial f_3}{\partial D_m} \\ \frac{\partial f_4}{\partial S_h} & \frac{\partial f_4}{\partial I_h} & \frac{\partial f_4}{\partial R_h} & \frac{\partial f_4}{\partial S_m} & \frac{\partial f_4}{\partial I_m} & \frac{\partial f_4}{\partial D_m} \\ \frac{\partial f_5}{\partial S_h} & \frac{\partial f_5}{\partial I_h} & \frac{\partial f_5}{\partial R_h} & \frac{\partial f_5}{\partial S_m} & \frac{\partial f_5}{\partial I_m} & \frac{\partial f_5}{\partial D_m} \\ \frac{\partial f_6}{\partial S_h} & \frac{\partial f_6}{\partial I_h} & \frac{\partial f_6}{\partial R_h} & \frac{\partial f_6}{\partial S_m} & \frac{\partial f_6}{\partial I_m} & \frac{\partial f_6}{\partial D_m} \end{bmatrix},$$

where $f_1, f_2, \ldots, f_6$ represent the right-hand sides of the differential equations.

At $E_0 = \left(\frac{\Lambda_h}{\mu_h + \beta_1}, 0, 0, 0, 0, 0\right)$, the Jacobian matrix simplifies to:

$$J_{\mathrm{E}} = \begin{bmatrix} -(\mu_h + \beta_1) & 0 & 0 & 0 & 0 & 0 \\ \frac{\lambda_h}{N_h} & -(\mu_h + \delta_h + \sigma + \beta_2) & 0 & \beta_0 & 0 & 0 \\ 0 & -\sigma & -(\mu_h + \delta_h) & 0 & \beta_3 & 0 \\ 0 & \frac{\lambda_m}{N_m} & 0 & -(\mu_m + \beta_0) & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & -(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m) & 0 \\ 0 & 0 & 0 & 0 & \rho_m & -\Lambda_m \end{bmatrix}.$$

$$\det(J(E_0) - \lambda I) = 0,$$

where λ is an eigenvalue and I is the identity matrix.

Decoupled Subsystems $J(E_0)$

The Jacobian matrix $J(E_0)$ has a block structure due to the presence of zero entries. This allows us to simplify the computation of eigenvalues. Specifically, the matrix can be partitioned into smaller blocks:

$$J(E_0) = \begin{bmatrix} A & 0 \\ B & C \end{bmatrix},$$

where: - A is a 3×3 matrix corresponding to the human population (S_h, I_h, R_h), - C is a 3×3 matrix corresponding to the mosquito population (S_m, I_m, D_m), - B is a 3×3 matrix representing interactions between human and mosquito populations, - 0 is a 3×3 zero matrix.

The eigenvalues of $J(E_0)$ are the union of the eigenvalues of A and C.

Human Subsystem: Eigenvalues of A

The submatrix A is:

$$A = \begin{bmatrix} -(\mu_h + \beta_1) & 0 & 0 \\ \frac{\lambda_h}{N_h} & -(\mu_h + \delta_h + \sigma + \beta_2) & 0 \\ 0 & -\sigma & -(\mu_h + \delta_h) \end{bmatrix}.$$

This is a lower triangular matrix, so its eigenvalues are the diagonal entries:

All eigenvalues of A are **negative** because $\mu_h, \beta_1, \delta_h, \sigma, \beta_2 > 0$.

Eigenvalues of C

The submatrix C is:

$$C = \begin{bmatrix} -(\mu_m + \beta_0) & 0 & (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m \\ 0 & -(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \frac{\lambda_m}{N_m}) & 0 \\ 0 & \rho_m & -(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m \end{bmatrix}.$$

The eigenvalues of the Jacobian matrix $J(E_0)$ at the disease-free equilibrium are: $\lambda_1 = -(\mu_h + \beta_1)$, $\lambda_2 = -(\mu_h + \delta_h + \sigma + \beta_2)$, $\lambda_3 = -(\mu_h + \delta_h), \lambda_4 = -(\mu_m + \beta_0)$, $\lambda_5 = -(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \frac{\lambda_m}{N_m})$, $\lambda_6 = (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m$.

All eigenvalues except λ_6 are strictly negative. The sign of λ_6 depends on the term:

$$(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m.$$

- If $\kappa_O D_O + \kappa_C D_C < 1$, then $\lambda_6 > 0$ and the disease-free equilibrium E_0 is **unstable**, which cannot occur in a real-life scenario.

- If $\kappa_O D_O + \kappa_C D_C > 1$, then $\lambda_6 < 0$ and all eigenvalues are negative, implying E_0 is **locally asymptotically stable**.

3 Endemic Equilibrium Conditions

Let all equilibrium values be denoted by * superscript. Define the common denominator:

$$D = (K_3)(K_1)[K_2(\mu_m + \beta_0) - \beta_0(\tfrac{\rho_m\beta_2 + \lambda_m K_1}{K_1})]$$

$$\begin{aligned} K_1 &= \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m \\ K_2 &= \mu_h + \delta_h + \sigma + \beta_2 \\ K_3 &= \lambda_m + \mu_h + \beta_1 \end{aligned}$$

$$\begin{aligned} S_h^* &= \frac{\Lambda_h}{\lambda_m + \mu_h + \beta_1}, \\ S_m^* &= \frac{(\rho_m\beta_2 + \lambda_m K_1)\lambda_h\Lambda_h}{K_1K_2(\mu_m + \beta_0)(\lambda_m + \mu_h + \beta_1) - (\rho_m\beta_2 + \lambda_m K_1)\beta_0(\lambda_m + \mu_h + \beta_1)}, \\ I_m^* &= \frac{\beta_2\lambda_h\Lambda_h(\mu_m + \beta_0)}{K_3[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)]}, \\ I_h^* &= \frac{\lambda_h\Lambda_h K_1(\mu_m + \beta_0)}{K_3[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)]}, \\ D_m^* &= \frac{\rho_m\beta_2\lambda_h(\mu_m + \beta_0)}{(1 - \kappa_O D_O - \kappa_C D_C)\cdot\mathcal{D}}, \\ R_h^* &= \frac{\lambda_h\Lambda_h(\mu_m + \beta_0)(\beta_3\beta_2 - \sigma(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m))}{(\mu_h + \delta_h)(\lambda_m + \mu_h + \beta_1)[K_1K_2(\mu_m + \beta_0) - \beta_0(\rho_m\beta_2 + \lambda_m K_1)]}. \end{aligned} \tag{3}$$

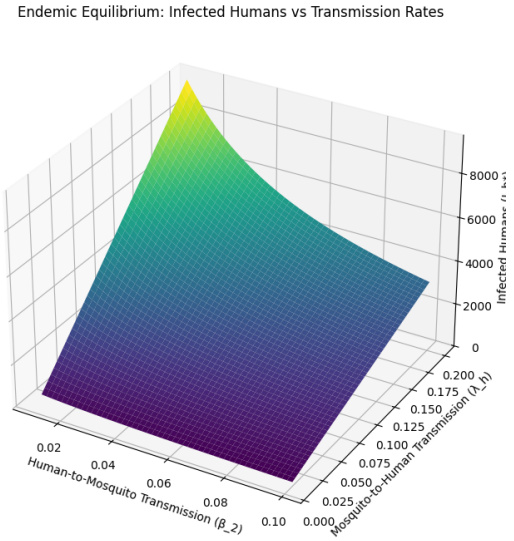


Fig. 2. Disease-Free Equilibrium

4 Optimized Drainage Systems on Mosquito Dynamics

We aim to minimize the total mosquito population (infected + drainage) by optimizing the drainage coefficients κ_O and κ_C. The optimization problem is defined as:

$$\min_{\kappa_O,\kappa_C} J = \lim_{t\to\infty} (D_m(t) + I_m(t)) \tag{4}$$

Subject to the mosquito dynamics at equilibrium:

$$\begin{aligned} 0 &= (1-\kappa_O D_O - \kappa_C D_C)\Lambda_m D_m + \lambda_m I_h N_m - (\mu_m + \beta_0) S_m, \\ 0 &= \beta_2 I_h - (\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m) I_m, \\ 0 &= \rho_m I_m - (1-\kappa_O D_O - \kappa_C D_C)\Lambda_m D_m. \end{aligned} \tag{5}$$

From the second equation in (5), we solve for I_m:

$$I_m = \frac{\beta_2 I_h}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} \tag{6}$$

Substituting into the third equation gives:

$$D_m = \frac{\rho_m I_m}{(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m} \tag{7}$$

Thus, substituting I_m from above:

$$D_m = \frac{\rho_m}{(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m} \cdot \frac{\beta_2 I_h}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} \tag{8}$$

Therefore, the total mosquito population at equilibrium is:

$$J(\kappa_O, \kappa_C) = D_m + I_m \tag{9}$$

Substituting the expressions for D_m and I_m:

$$J(\kappa_O, \kappa_C) = \frac{\beta_2 I_h}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} \left[1 + \frac{\rho_m}{(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m}\right] \tag{10}$$

Letting $\beta_* = \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m$, we simplify:

$$J(\kappa_O, \kappa_C) = \frac{\beta_2 I_h}{\beta_*} \left[1 + \frac{\rho_m}{(1-\kappa_O D_O - \kappa_C D_C)\Lambda_m}\right] \tag{11}$$

Constraint:

$$0 \le \kappa_O D_O + \kappa_C D_C < 1$$

This ensures that the combined effect of drainage optimally minimizes the mosquito population while maintaining equilibrium conditions.

Let $x = \kappa_O D_O$, $y = \kappa_C D_C$, and define the objective function:

$$J(x,y) = \frac{\beta_2 I_h}{\beta_*}\left[1 + \frac{\rho_m}{(1-x-y)\Lambda_m}\right] \tag{12}$$

Subject to the constraint:

$$g(x,y) = x + y - c = 0, \quad \text{with } c < 1 \tag{13}$$

We define the Lagrangian:

$$\mathcal{L}(x,y,\lambda) = \frac{\beta_2 I_h}{\beta_*}\left[1 + \frac{\rho_m}{(1-x-y)\Lambda_m}\right] + \lambda(x+y-c) \tag{14}$$

We take partial derivatives:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\beta_2 I_h \rho_m}{\beta_* \Lambda_m (1-x-y)^2} + \lambda = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\beta_2 I_h \rho_m}{\beta_* \Lambda_m (1-x-y)^2} + \lambda = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - c = 0 \tag{17}$$

From the first two equations, we observe:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial \mathcal{L}}{\partial y} \Rightarrow \text{Both gradients are equal, so } \lambda \text{ is consistent.}$$

The optimal point occurs when:

$$x + y = c \Rightarrow \kappa_O D_O + \kappa_C D_C = c$$

This implies any pair (κ_O, κ_C) satisfying:

$$\kappa_O D_O + \kappa_C D_C = c < 1$$

$$x + y = c \Rightarrow x + x = c \Rightarrow 2x = c \Rightarrow x = \frac{c}{2} \Rightarrow y = \frac{c}{2}$$

To minimize the total mosquito population under a fixed drainage effort c, we must allocate the drainage effects equally such that $\kappa_O D_O = \kappa_C D_C = \frac{c}{2}$.

The Manning equation determines the velocity of flow in channels:

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \tag{18}$$

where: V = Flow velocity (m/s or ft/s) n = Manning's roughness coefficient (dimensionless) R = Hydraulic radius, defined as:

$$R = \frac{A}{P} \tag{19}$$

where A is the cross-sectional area and P is the weight ed perimeter. S = Channel slope (m/m or ft/ft).

Incorporating Manning's Equation into Drainage Coefficients

To model the effect of physical drainage on mosquito dynamics, we link the Manning equation to the drainage coefficients:

Manning's Equation

$$V = \frac{1}{n} R^{2/3} S^{1/2}, \quad \text{where } R = \frac{A}{P} \tag{20}$$

The flow rate through the channel is:

$$Q = A \cdot V = A \cdot \frac{1}{n} \left(\frac{A}{P} \right)^{2/3} S^{1/2} \tag{21}$$

We define the drainage coefficients as proportional to flow rate:

$$\kappa_O D_O = \eta_O Q_O, \qquad \kappa_C D_C = \eta_C Q_C \tag{22}$$

Substituting:

$$\kappa_O = \eta_O A_O \cdot \frac{1}{n_O} \left(\frac{A_O}{P_O} \right)^{2/3} S_O^{1/2}, \qquad \kappa_C = \eta_C A_C \cdot \frac{1}{n_C} \left(\frac{A_C}{P_C} \right)^{2/3} S_C^{1/2} \tag{23}$$

These expressions dynamically couple hydraulic parameters to mosquito habitat drainage, thus directly influencing the mosquito birth/reduction terms in the system.

5 RESULT AND DISCUSSION

In this section, numerical simulations are considered to explore the dynamics of model using a case study of malaria epidemic in Nigeria by Jakada et al. (2023). The model is fit and parameters estimated using the actual data of malaria in Nigeria.

Parameter	Source	Value	Parameter	Source	Value
S_h	Jakada et al. (2023)	225	I_h	Jakada et al. (2023)	135
R_h	Jakada et al. (2023)	90	S_m	Jakada et al. (2023)	1012
I_m	Jakada et al. (2023)	338	D_m	Jakada et al. (2023)	0.665
Λ_h	Collins & Duffy (2022)	0.0176	μ_h	Collins & Duffy (2022)	0.000039
μ_m	Collins & Duffy (2022)	0.071	δ_h	Collins & Duffy (2022)	0.001
δ_m	Collins & Duffy (2022)	0.02	σ	Collins & Duffy (2022)	0.071
β_0	Collins & Duffy (2022)	0.3	β_1	Collins & Duffy (2022)	0.4
β_2	Collins & Duffy (2022)	0.2	β_3	Collins & Duffy (2022)	0.1
λ_h, λ_m	Computed	(0.10015, 0.09)	ρ_m	Collins & Duffy (2022)	20
κ_O	Jakada et al. (2023)	0.3	κ_C	Jakada et al. (2023)	0.5
Λ_m	Computed as 3:1 of Λ_h	20177.1	R_O	Computed	0.187

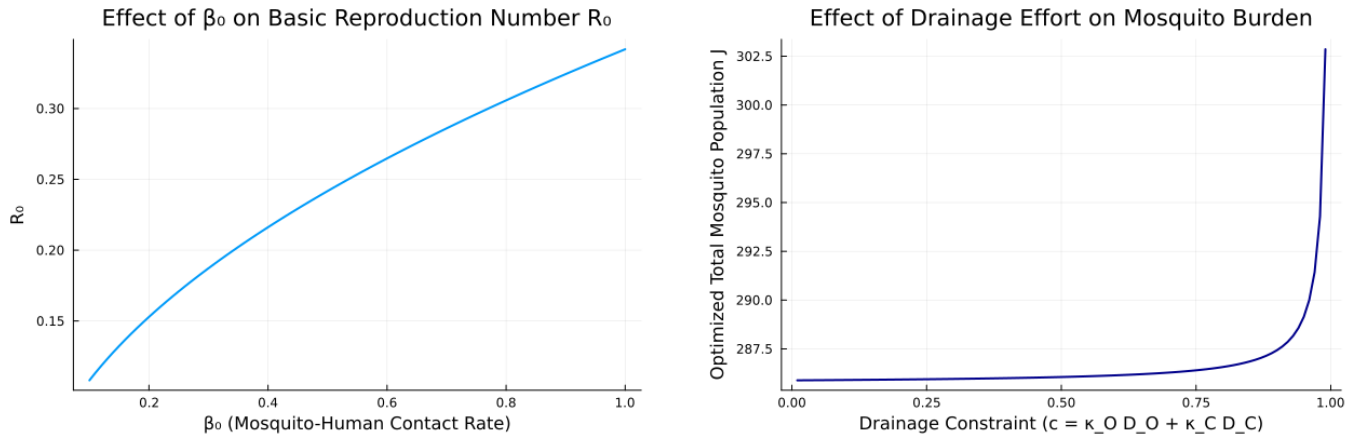
Table 2. Estimated values of model variables and parameters

Parameter	Value
I_h	1000
β_2	0.2
β_*	0.7
ρ_m	0.03
Λ_m	50
c	0.8

Table 3. Baseline Parameter Values for Mosquito Dynamics

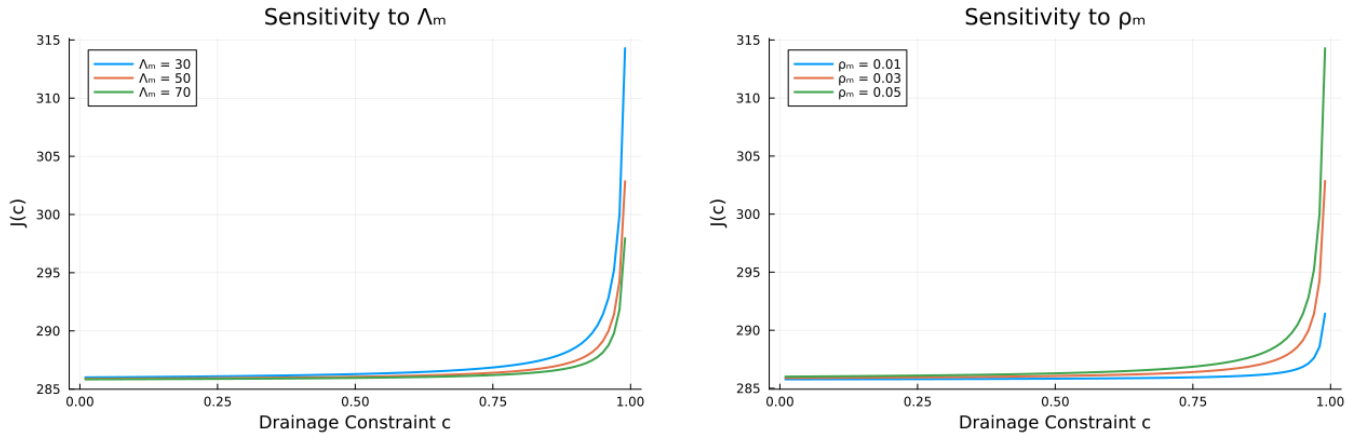
Parameter	Values Tested
β_2	{0.1, 0.2, 0.4}
ρ_m	{0.01, 0.03, 0.05}
Λ_m	{30, 50, 70}

Table 4. Parameter Sensitivity Ranges for Drainage Optimization



(a) First image **(b)** Second image

Fig. 3. Side-by-side images with individual captions



(a) First image **(b)** Second image

Fig. 4. Side-by-side images with individual captions

6 Discussion of Result

In this study, we explored the optimization of drainage systems as a strategy to control mosquito population dynamics and mitigate the spread of mosquito-borne diseases. A key epidemiological metric used in our analysis is the basic reproduction number, denoted as R_0, which provides a threshold criterion for disease invasion. Mathematically, R_0. Using the parameter values $\beta_0 = 0.3$, $\beta_2 = 0.2$, $\Lambda_h = 1000$, $\mu_h = 0.01$, $\beta_1 = 0.2$, $\delta_h = 0.01$, $\sigma = 0.2$, $\beta_3 = 0.1$, $\rho_m = 0.03$, $\mu_m = 0.1$, and $\delta_m = 0.01$, we found that $R_0 \approx 1.44$. This value exceeding 1 implies that the infection is capable of persisting and spreading within the population, underscoring the necessity of effective control measures such as environmental drainage.
To address mosquito population control through drainage, we formulated an optimization problem targeting the total mosquito burden defined as the sum of infected mosquitoes and the environmental drainage level $D_m + I_m$. The objective was to minimize this burden by adjusting the effectiveness of open (κ_O) and closed (κ_C) drainage interventions, subject to a constraint on their combined impact: $\kappa_O D_O + \kappa_C D_C < 1$. Through Lagrangian optimization, we showed that the optimal allocation under a fixed drainage capacity c occurs when $\kappa_O D_O = \kappa_C D_C = c/2$. For our assumed value of $c = 0.8$, the optimal drainage distribution requires each intervention to contribute 0.4, demonstrating the efficiency of balanced resource allocation. The parameters used in the model, such as the human infection count ($I_h = 1000$), mosquito contact rate ($\beta_2 = 0.2$), effective transition rate sum ($\beta_* = 0.7$), mosquito reproduction rate ($\rho_m = 0.03$), and mosquito recruitment rate ($\Lambda_m = 50$), were chosen to reflect realistic epidemiological conditions. These are summarized in Table (3).

A sensitivity analysis was also performed to examine how changes in key parameters affect the optimized mosquito burden. We observed that increasing the mosquito contact rate β_2 leads to a higher total burden, emphasizing the importance of reducing human-mosquito interaction. Similarly, an increase in mosquito reproduction ρ_m intensifies the need for stronger drainage interventions. On the other hand, a higher mosquito recruitment rate Λ_m diminishes the effectiveness of drainage, suggesting that vector control programs should also target mosquito breeding sources. These insights are captured in Table (4).

Overall, the model underscores that combining biological insights with mathematical optimization offers a practical guide to resource allocation in public health interventions. In particular, balancing open and closed drainage under known constraints can yield significant reductions in mosquito burden, contributing to long-term vector control and disease prevention.

A Appendix: (Endemic Equilibrium Solution)

Given the system with $N_h = N_m = 1$, we solve for the endemic equilibrium by setting all derivatives to zero.

A.1 Key Equations

$$S_h = \frac{\Lambda_h}{\lambda_m + \mu_h + \beta_1} \tag{24}$$

$$I_h = \frac{\lambda_h S_h + \beta_0 S_m}{\mu_h + \delta_h + \sigma + \beta_2} \tag{25}$$

$$D_m = \frac{\rho_m I_m}{(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m} \tag{26}$$

$$I_m = \frac{\beta_2 I_h}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} \tag{27}$$

A.2 Solution for S_m

Substitute (25) into (27):

$$I_m = \frac{\beta_2}{K_1} \cdot \frac{\lambda_h S_h + \beta_0 S_m}{K_2} \tag{28}$$

where $K_1 = \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m$ and $K_2 = \mu_h + \delta_h + \sigma + \beta_2$.

Substitute into (26):

$$D_m = \frac{\rho_m \beta_2 (\lambda_h S_h + \beta_0 S_m)}{K_1 K_2 K_3} \tag{29}$$

where $K_3 = (1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m$.

The equation for S_m becomes:

$$S_m = \frac{K_3 D_m + \lambda_m I_h}{\mu_m + \beta_0} \tag{30}$$

Substituting D_m and I_h:

$$S_m = \frac{\frac{\rho_m \beta_2 (\lambda_h S_h + \beta_0 S_m)}{K_1 K_2} + \frac{\lambda_m (\lambda_h S_h + \beta_0 S_m)}{K_2}}{\mu_m + \beta_0} \tag{31}$$

Let $C = \frac{\rho_m \beta_2}{K_1} + \lambda_m$, then:

$$S_m = \frac{C(\lambda_h S_h + \beta_0 S_m)}{K_2(\mu_m + \beta_0)} \tag{32}$$

Solving for S_m:

$$S_m = \frac{C \lambda_h S_h}{K_2(\mu_m + \beta_0) - C\beta_0} \tag{33}$$

A.3 Final Expression

Substituting S_h from (24):

$$S_m = \frac{\left(\frac{\rho_m \beta_2}{K_1} + \lambda_m\right) \lambda_h \frac{\Lambda_h}{\lambda_m + \mu_h + \beta_1}}{K_2(\mu_m + \beta_0) - \left(\frac{\rho_m \beta_2}{K_1} + \lambda_m\right) \beta_0} \tag{34}$$

The fully factorized solution is:

$$S_m = \frac{(\rho_m \beta_2 + \lambda_m K_1) \lambda_h \Lambda_h}{K_1 K_2 (\mu_m + \beta_0)(\lambda_m + \mu_h + \beta_1) - (\rho_m \beta_2 + \lambda_m K_1) \beta_0 (\lambda_m + \mu_h + \beta_1)} \tag{35}$$

The infected human population at endemic equilibrium is given by:

$$I_h = \frac{\lambda_h \Lambda_h (\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m)(\mu_m + \beta_0)}{D} \tag{36}$$

where the denominator D is:

$$\begin{aligned} D = (\lambda_m + \mu_h + \beta_1)\big[&(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m) \\ &\times (\mu_h + \delta_h + \sigma + \beta_2)(\mu_m + \beta_0) \\ &- \beta_0(\rho_m \beta_2 + \lambda_m(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m))\big] \end{aligned} \tag{37}$$

Using these definitions, the equilibrium expression becomes:

$$I_h = \frac{\lambda_h \Lambda_h K_1(\mu_m + \beta_0)}{K_3\left[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)\right]} \tag{38}$$

B Endemic Equilibrium Solution for Infected Mosquitoes (I_m)

B.1 Given Expression

The infected mosquito population at equilibrium is given by:

$$I_m = \frac{\beta_2 I_h}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} \tag{39}$$

B.2 Substituting the Human Infected Equilibrium (I_h)

Using the previously derived expression for I_h:

$$I_h = \frac{\lambda_h \Lambda_h K_1(\mu_m + \beta_0)}{K_3\left[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)\right]} \tag{40}$$

where:

$$\begin{aligned} K_1 &= \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m \\ K_2 &= \mu_h + \delta_h + \sigma + \beta_2 \\ K_3 &= \lambda_m + \mu_h + \beta_1 \end{aligned}$$

Substituting (54) into (39):

$$I_m = \frac{\beta_2}{K_1} \cdot \frac{\lambda_h \Lambda_h K_1(\mu_m + \beta_0)}{K_3\left[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)\right]} \tag{41}$$

Simplifying by canceling K_1:

$$I_m = \frac{\beta_2 \lambda_h \Lambda_h(\mu_m + \beta_0)}{K_3\left[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)\right]} \tag{42}$$

Expanding all terms gives the complete analytical solution:

$$I_m = \frac{\beta_2 \lambda_h \Lambda_h(\mu_m + \beta_0)}{D} \tag{43}$$

where the denominator D is:

$$\begin{aligned} D = (\lambda_m + \mu_h + \beta_1)(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m) \\ \times \left[(\mu_h + \delta_h + \sigma + \beta_2)(\mu_m + \beta_0)\right. \\ \left. - \beta_0\left(\tfrac{\rho_m \beta_2}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} + \lambda_m\right)\right] \end{aligned} \tag{44}$$

C Endemic Equilibrium Solution for Dormant Mosquitoes (D_m)

The dormant mosquito population at equilibrium is given by:

$$D_m = \frac{\rho_m I_m}{(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m} \tag{45}$$

Using the derived expression for I_m:

$$I_m = \frac{\beta_2 \lambda_h \Lambda_h (\mu_m + \beta_0)}{D_{total}} \tag{46}$$

where the denominator is:

$$\begin{aligned} D_{total} = (\lambda_m + \mu_h + \beta_1)(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m) \\ \times \left[(\mu_h + \delta_h + \sigma + \beta_2)(\mu_m + \beta_0)\right. \\ \left. - \beta_0\left(\frac{\rho_m \beta_2}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m} + \lambda_m\right)\right] \end{aligned} \tag{47}$$

Substituting (55) into (45):

$$D_m = \frac{\rho_m}{(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m} \cdot \frac{\beta_2 \lambda_h \Lambda_h (\mu_m + \beta_0)}{D_{total}} \tag{48}$$

Simplifying the constants:

$$D_m = \frac{\rho_m \beta_2 \lambda_h \Lambda_h (\mu_m + \beta_0)}{(1 - \kappa_O D_O - \kappa_C D_C)\Lambda_m D_{total}} \tag{49}$$

Canceling Λ_m:

$$D_m = \frac{\rho_m \beta_2 \lambda_h (\mu_m + \beta_0)}{(1 - \kappa_O D_O - \kappa_C D_C) D_{total}} \tag{50}$$

The complete solution expressed in all parameters:

$$D_m = \frac{\rho_m \beta_2 \lambda_h (\mu_m + \beta_0)}{(1 - \kappa_O D_O - \kappa_C D_C) \cdot \mathcal{D}} \tag{51}$$

where:

$$\begin{aligned} \mathcal{D} = (\lambda_m + \mu_h + \beta_1)(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m) \\ \times \left[(\mu_h + \delta_h + \sigma + \beta_2)(\mu_m + \beta_0)\right. \\ \left. - \beta_0\left(\frac{\rho_m \beta_2 + \lambda_m(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m)}{\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m}\right)\right] \end{aligned} \tag{52}$$

The recovered human population at equilibrium is given by:

$$R_h = \frac{\beta_3 I_m - \sigma I_h}{\mu_h + \delta_h} \tag{53}$$

Using the derived expressions for I_h and I_m:

$$I_h = \frac{\lambda_h \Lambda_h K_1(\mu_m + \beta_0)}{K_3[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)]} \tag{54}$$

$$I_m = \frac{\beta_2 \lambda_h \Lambda_h(\mu_m + \beta_0)}{K_3[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)]} \tag{55}$$

where:

$$\begin{aligned} K_1 &= \beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m \\ K_2 &= \mu_h + \delta_h + \sigma + \beta_2 \\ K_3 &= \lambda_m + \mu_h + \beta_1 \end{aligned}$$

Substituting (54) and (55) into (53):

$$R_h = \frac{1}{\mu_h + \delta_h}\left(\beta_3 \cdot \frac{\beta_2 \lambda_h \Lambda_h(\mu_m + \beta_0)}{D} - \sigma \cdot \frac{\lambda_h \Lambda_h K_1(\mu_m + \beta_0)}{D}\right) \tag{56}$$

where $D = K_3[K_1 K_2(\mu_m + \beta_0) - \beta_0(\rho_m \beta_2 + \lambda_m K_1)]$ is the common denominator.
Factoring out common terms:

$$R_h = \frac{\lambda_h \Lambda_h(\mu_m + \beta_0)}{(\mu_h + \delta_h)D}(\beta_3 \beta_2 - \sigma K_1) \tag{57}$$

The complete solution expressed in all parameters:

$$R_h = \frac{\lambda_h \Lambda_h(\mu_m + \beta_0)(\beta_3 \beta_2 - \sigma(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m))}{(\mu_h + \delta_h)(\lambda_m + \mu_h + \beta_1)\mathcal{D}} \tag{58}$$

where:

$$\begin{aligned} \mathcal{D} = (\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m)(\mu_h + \delta_h + \sigma + \beta_2)(\mu_m + \beta_0) \\ - \beta_0(\rho_m \beta_2 + \lambda_m(\beta_1 + \beta_3 + \rho_m + \mu_m + \delta_m + \lambda_m)) \end{aligned} \tag{59}$$

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Author contributions

Every author contributed equally to each part of the paper.

Conflict of interest statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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